ISSN 2079–1836
2018, вип. 28

UDC 531/534.573

O. V. Pogrebnoy

ON MODELING OF TUBULAR BONE CROSS-SECTION FOR DESIGNING ENDOPROSTHESIS STEM

Modeling of cross-section contour of bone marrow channel is considered. Different methods of spline approximation are studied in common case. An approach for modeling of cross-section with characteristic form on the base of two curves combining is suggested in particular case. A method for contour description with varying one parameter of endoprosthesis stem is analyzed. Possibility of press-fit an endoprosthesis stem into marrow channel is examined. The principles of endoprothesis stem design on the base of 3D-model of bone marrow channel are proposed.

Keywords: *modeling, approximation, spline, multispiral computer tomography, tubular bone, bone marrow channel, endoprosthesis.*

Introduction. Recovery of move functional range is the aim of treatment of finger injured joints. The effectiveness of wrist rehabilitation depends on a choice of the method and tactics of operative treatment. The correct choice leads to an earlier functional reconstruction of limb and invalidity shortening [1].

New approaches for treatment of consequences of joint finger wrist traumas are introduced in surgery. Development of endoprosthesis methods discovers new possibilities of patient rehabilitation such as loss function reconstruction of the wrist after posttraumatic defects and joint finger deformation [2].

For design and manufacturing of individual endoprosthesis it is necessary to examine a form of marrow bone channel. An approach for working out of channel 3D-model on the base of multispiral computer tomography is described in [2, 3]. More precise approach for 3D-model construction is given in [4]. It can be possible that 3D-model has curvatures both in the sagittal and in frontal planes.

Purpose of paper is a development of an approach for design of endoprosthesis stem using 3D-model of marrow bone channel. The angles of cross-sections have to be smoothed for receiving a smooth lateral surface. Sizes of cross-section have to increase monotonically from the tip to the base of endoprosthesis stem.

Principles for designing of model of endoprosthesis stem. Modeling of marrow bone channel cross-section and subsequent design of individual endoprosthesis stem has to be founded on following principles.

1. Minimization of bone tissue.

2. Minimization of free volume between stem cross-section and marrow bone channel cross-section.

3. Conservation of cortical bone wall width.

4. Virtual curvature of endoprosthesis stem only in the sagittal plane.

5. Sizes of cross-section have to increase monotonically from the tip to the base of endoprosthesis stem.

6. Additional possibility to realize a fit-press of stem into marrow bone channel with specified a priori measure.

Implementation into practice. In common case (arbitrary configuration of cross-section) it is proposed to use spline approximation for modeling the contour of cross-section. In particular case with cross-section similar to the round a usage of equation of circle or ellipse is convenient. The proximal phalanx cross-sections have a character form with convex vault and convex, concave or even basement. In this case an application of proposed in this article combined figure seems to be rational.

In all the cases the coordinates of the centre of gravity to be calculated and an axis of endoprosthesis stem will be obtained. In common case it is proposed to find the centre of gravity with the use of analysis of digital image that contains a result of spline approximation. In particular cases the centre of gravity is found using the equations of curves that compose a geometrical figure.

Let consider some suitable methods for modeling of contour by means of spline approximation. An example of cross-section of II bone metacarpus is given on Fig. 1. The contour of marrow bone channel is marked with system of boundary points. The very simple method is to unite the border points by pieces of straight line: piecewise-linear approximation. An approach realization is simple too, the created figure is similar to the real cross-section, but it is possible a presence of acute angles, Fig. 2.

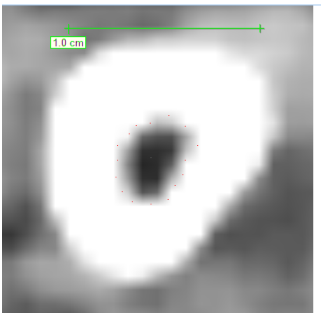

Fig. 1 – **Original image of cross-section**

The result of usage of B-spline (Bézier splines) approximation is shown on Fig. 3. The smooth curve is built, but it is obvious some decrease of contour area.

Cubic spline approximation leads to convenient result (Fig. 4). All the border points are included in approximation line, though insignificant exceeding of cross-section area is also presented. The result of applying of the composite Bézier splines is shown in Fig. 5. Apart points are situated out of approximation curve, general form of cross-section is preserved, but some diminution of cross-section area is obvious.

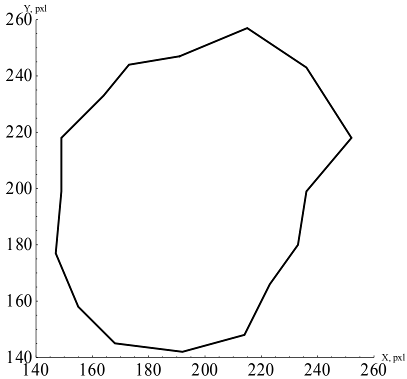

Fig. 2 – **Piecewise approximation**

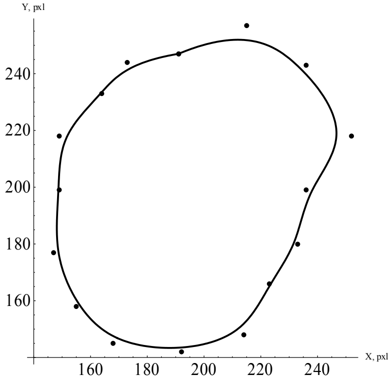

Fig. 3 – **B-spline approximation**

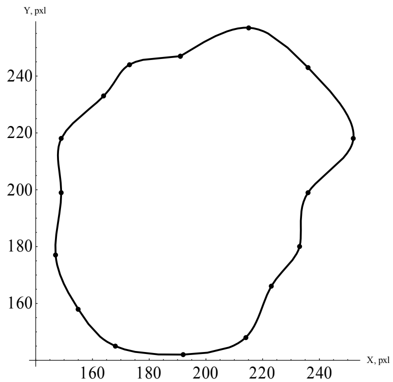

Fig. 4 – **Cubic-spline approximation**

Bézier splines

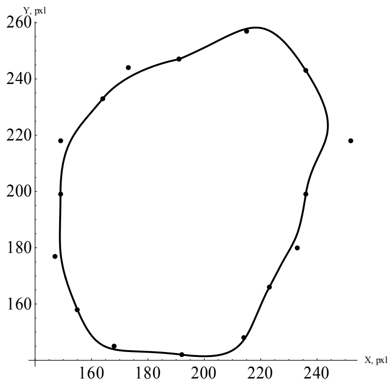

Fig. 5 – **Approximation**

with the composite

Comparison of results of presented approximation methods is given in Table 1. The area of piecewise approximation is taken as base for calculation.

Table 1 – **Coordinates of the centre of gravity and area of cross-section**

№	**Approximation method**	**Coord.** x	**Coord.** y	**Pixels**	**Area, %**
1	Piecewise linear	196,09	201,98	8392	100
2	Bézier splines	195,91	202,39	8131	96,89
3	Cubic splines	196,44	201,80	8707	103,75
4	Composite Bézier splines	195,82	202,61	8355	99,56

As it seen from Tabl. 1, the differences in centre of gravity coordinates are negligible (maximum 0,64 pixels or 0,025 mm). The differences in cross-section areas do not exceed 4%.

Let consider particular case of cross-section form that is characteristic for proximal phalanx of digits. Image on Fig. 6 is made with use of the multispiral computer tomography. Cross-sections are marked by rectangles and consist of convex vault and some plane base that may have also some convexity or concavity.

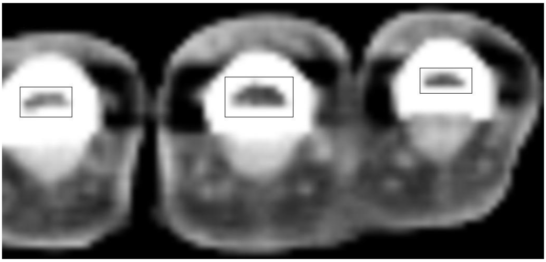

Fig. 6 – **Cross-sections of proximal phalanx of fingers**

For description of cross-sections of such type a special figure is proposed. It consists from two parts: upper (vault) is formed by an elongated cycloid and lower (base) is a shortened cycloid or a parabola. The shortened cycloid is used for concave base and a parabola for convex or plane one. The necessary condition for creating this figure is the smooth conjugation between both upper and lower parts.

With the use of combining of elongated and shortened cycloids (Fig. 7) is formed the figure with concaved base, Fig. 8.

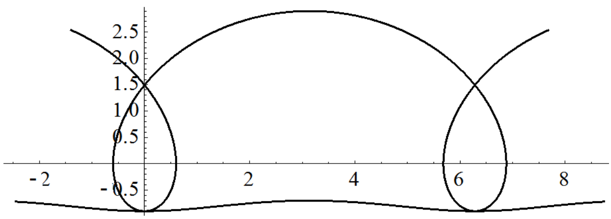

Fig. 7 – **Elongated and shortened cycloids**

Parts on Fig. 8 formed by elongated and shortened cycloids are marked with different filling and hatch density. The smooth conjugation of both curves in the lower points of figure is achieved.

The contour of cross-section with plane or convex base is proposed to describe by a figure combined of elongated cycloid (upper part) and a parabola (lower part) by reason of its good conjunction (Fig. 9).

The parabola coefficients are adjusted by the least-square method under condition of fitting of coordinates of two lower cycloid points and the lower point of base to the parabola equation. The combined figure is presented on Fig. 10, the position of centre of gravity is marked with small cross.

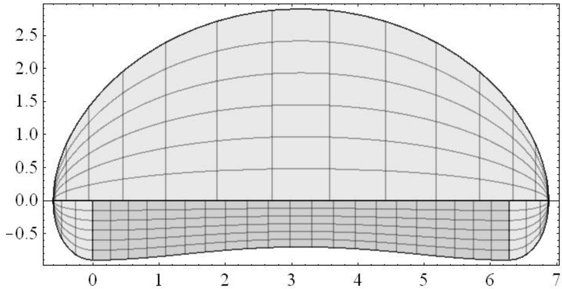

Fig. 8 – **Combined figure for modeling the cross-section with concaved base**

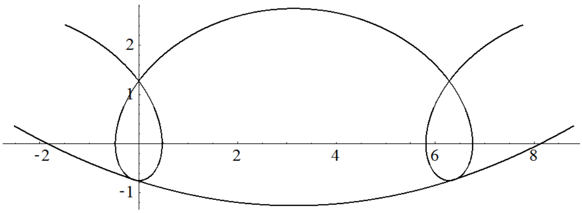

Fig. 9 – **Elongated cycloid and parabola**

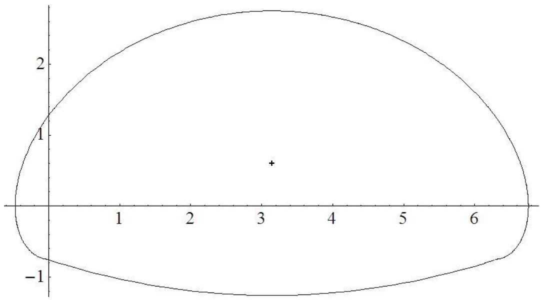

Fig. 10 - **Combined figure for modeling bi-convex cross-section**

In both cases the coordinates of centre of gravity are calculated using the positions of centre of gravity of two parts. Later the positions of cross-section centre of gravity are used for creation of the endoprosthesis stem axis.

Thus, two approaches for modeling of marrow bone channel cross-section are described. First one is based on the approximation methods in arbitrary positions of boundary points (common case). Second approach may be useful for the cross-section with special form (particular case).

Test example. Consider a specific example of using mentioned above principles 1-3 for modeling of cross-section. System of boundary points that formed a rounded contour is shown on Fig. 11. Points are numbered from 1 to 22 and united with straight lines. The area of polygon and coordinates of the centre of gravity are calculated too. The polygon is divided into system of triangles. For modeling the rounded contour a usage of circle is convenient.

The centre of circle is situated in the centre of gravity of the polygon. The circle with radius that equals to a distance to the nearest point we consider as the circle with minimum radius. Here the resection is equals to zero and the free area equals to maximum, 100%. The circle with maximum radius (distance to the very remote point) corresponds to maximum of resection and minimum of gap. It is obvious that on practice the minimal circle is unusable and the maximal circle is rather convenient. However a range of radius variety is determined. The minimal and maximal circles are shown on Fig. 11.

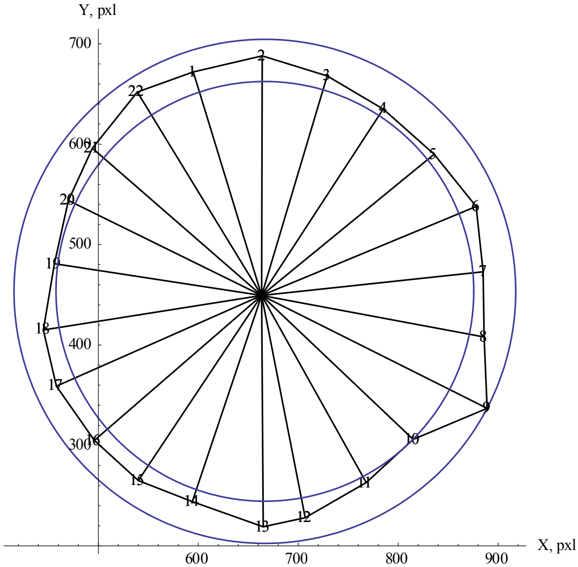

Fig. 11 – **System of boundary points, minimal and maximal circles**

The circle with an area that is equal to the polygon area forms the equality (balance) between resection and gaps. The system of boundary points and the circle with equal radius are presented on Fig. 12.

For fulfillment of principle 6 the calculation of the resection area and the area of gaps was made with radius variety in determined range. The resection area and the area of gaps between 9th and 10th boundary points are shown on Fig. 13. By chance in 9th point the radius equals to maximum and in 10th point equals to minimum. The resection area is denoted as *Rez* and the area of gaps as *Luft*.

The resection area S_R and the area of gaps S_L depending on radius R are presented on Fig. 14. The values S_R and S_L are normalized to the sum of its maximum values. The value R is measured in pixels and varying in range [209,18; 251,12]. The point of intersection of curves S_R and S_L corresponds to the equality (balance) of the resection area and the area of gaps.

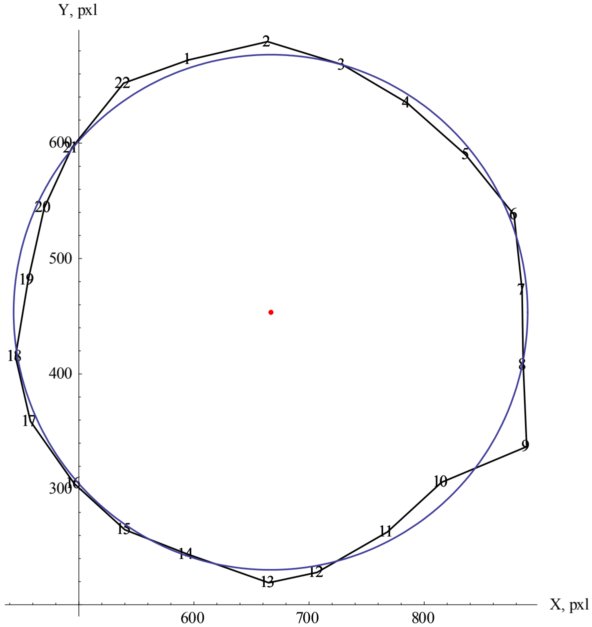

Fig. 12 - **System of boundary points and circle with equal radius**

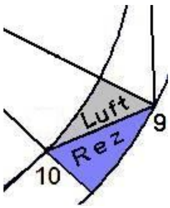

Fig. 13 – **The resection area and the area of gaps for 9th triangle**

In practice for endoprosthesis stable fixation it is quite possible to ensure some exceeding of the resection area over the area of gaps. The results presented on Fig. 14 give an opportunity to obtain the value of radius $\boldsymbol{R}$ when the resection area exceeds the area of gaps in some a priori specified value. Such value has to be determined on the basis of medical practice.

For this we consider the dependency of value $\boldsymbol{S_R} - \boldsymbol{S_L}$ on the radius $\boldsymbol{R}$ (Fig. 15). This dependency simultaneously describes an augmentation of the resection area $\boldsymbol{S_R}$ and a diminution of the area of gaps $\boldsymbol{S_L}$ i.e. characterizes a fit density of endoprosthesis stem into the marrow bone channel. On Fig. 15 the equation of approximation and the coefficient of approximation veracity $\boldsymbol{R^2}$ (do not mix with radius of circle $\boldsymbol{R}$) are presented.

Specifying, for example, a value $x = 25$ and after it substitution into the equation of approximation on Fig 15, we receive $y(x) = 239{,}442$ pixels, as shown on Fig. 16.

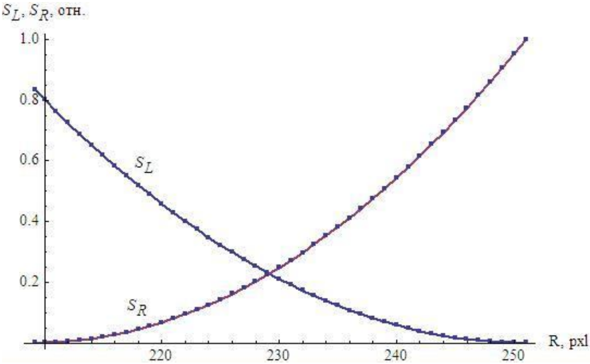

Fig. 14 – **Dependencies of the resection area and the area of gaps from radius**

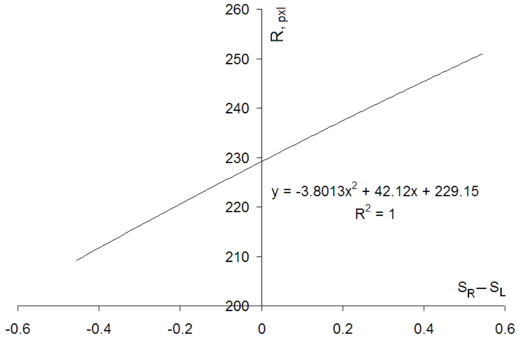

Fig. 15 – **Dependency of value $S_R - S_L$ from the radius R**

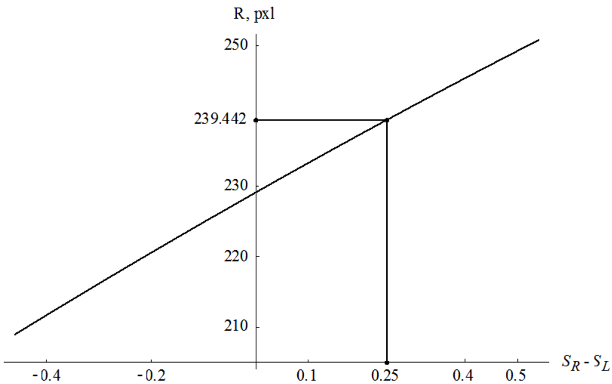

Fig. 16 – **Example for calculation of fit density of endoprosthesis stem**

After conversion from pixels to millimeters we receive the radius of endoprosthesis stem that ensures the specified press-fit density in considered cross-section.

Modeling of the cross-section using more complicated figures, such as ellipse or described above combined figure, needs to vary two parameters.

Conclusions. Methods of modeling of bone marrow channel cross-section with the use of spline approximation are considered. In particular case for the cross-section with characteristic form an approach on the base of two curves combining is suggested. Method for contour description with varying one parameter of endoprothesis stem is analyzed. Possibility of press-fit an endoprothesis stem into marrow bone channel is examined. Principles of endoprothesis stem design on the base of 3D-model of marrow bone channel are proposed.

Acknowledgements to Naumenko L. Yu., Dr. Sci. (Med.), and Mametiev A. A., PhD (Med.) for useful discuss of this paper.

REFERENCES

1. ***Naumenko L. Yu.***, *Mametiev A. A., Pogrebnoy O. V.* Mathematical modeling of marrow channel geometrical axis of hand long bones // Trauma. 2013. Vol. 14. No 5. P. 74–76. (in Russian).

2. ***Naumenko L. Yu.***, *Mametiev A. A., Pogrebnoy O. V.* Modeling of construction geometrical shape of inner joints fixing elements of implants // XVII Ukrainian congress of orthopedists and traumatologists, (Kiev, 5-7 octobre 2016.). 2016. P. 225–226. (in Russian).

3. ***Pogrebnoy O. V.*** Calculation of axis tubular bones marrow channel on the base of multispiral computer tomography // Problems of mechanics and strength of structures. Dnipropetrovsk: Lira. 2014. Vol. 22. P. 208–221. (in Russian).

4. ***Pogrebnoy O. V.*** Development of geometrical model of marrow bone channel of tubular bones on the base of multispiral computer tomography data // Problems of mechanics and strength of structures. Dnipropetrovsk: Lira. 2017. Vol. 24. P. 122–131. (in Russian).

УДК 531/534:57

О. В. Погребной

О МОДЕЛИРОВАНИИ ПОПЕРЕЧНОГО СЕЧЕНИЯ ТРУБЧАТЫХ КОСТЕЙ КИСТИ ДЛЯ ПРОЕКТИРОВАНИЯ НОЖКИ ЭНДОПРОТЕЗА

Рассмотрены способы моделирования контура поперечного сечения костно-мозгового канала трубчатых костей на основе аппроксимации сплайнами. Для частного случая поперечного сечения с характерной формой предложен способ моделирования его контура на основе сочетания двух кривых. Приведен способ описания контура поперечного сечения костно-мозгового канала, используя изменение формы сечения ножки эндопротеза по одному параметру. Рассмотрена возможность задания плотности посадки ножки эндопротеза. Изложены принципы проектирования ножки ендопротеза, исходя из 3D-модели реального костно-мозгового канала.

Ключевые слова: *моделирование, аппроксимация, сплайн, мультиспиральная компьютерная томография, трубчатая кость, костно-мозговой канал, ендопротез.*

УДК 531/534.573

О. В. Погрібний

ПРО МОДЕЛЮВАННЯ ПОПЕРЕЧНОГО ПЕРЕТИНУ ТРУБЧАСТИХ КІСТОК КИСТІ ДЛЯ ПРОЕКТУВАННЯ НІЖКИ ЕНДОПРОТЕЗУ

Розглянуто способи моделювання контура поперечного перетину кістково-мозкового каналу трубчастих кісток на основі апроксимації сплайнами. Для окремого випадку поперечного перетину з характерною формою запропоновано спосіб моделювання його контура на основі поєднання двох кривих. Наведено спосіб опису контуру поперечного перетину кістково-мозкового каналу, використовуючи зміну форми перетину ніжки ендопротезу по одному параметру. Розглянуто можливість завдання щільності посадки ніжки ендопротезу. Викладені принципи проектування ніжки ендопротезу, виходячи з 3D-моделі реального кістково-мозкового каналу.

Ключові слова: *моделювання, апроксимація, сплайн, мультиспиральна комп'ютерна томографія, трубчаста кістка, кістково-мозковий канал, ендопротез.*

Дніпровський національний
університет імені Олеся Гончара,
Дніпро, Україна

Надійшла до редколегії 10.09.2018