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VIBRATION OF SPACECRAFT STRUCTURE WITH JOINT-UP DYNAMIC ABSORBER AND PERIODIC DAMPING COEFFICIENT NEAR DISTURBED SURFACE

Two-step hybrid asymptotic approach on the basis of perturbation and phase-integral (or WKB) method is used for approximate analytical solution of nonlinear oscillation problem for spacecraft structure (with joined-up dynamic absorber and periodic in time damping coefficient) which moves near the disturbed surface to be discussed. Solution reduces itself to the necessity of integration of singular nonlinear (order *m*) differential equation with time-variant periodic coefficients at given initial conditions. Found approximate analytic solution is unbounded with the dimensionless value of disturbance range and degree of nonlinearity of restoring forces and is formed into sum that consists of two functions according to perturbation method (with respect to scalar parameter at nonlinear component of initial equation) and WKB-approximation (with respect to parameter near the highest derivative).

Keywords: *dynamics of spacecraft structure, disturbed surface, joined absorber, hybrid asymptotic approach, approximate analytical solution.*

Introduction. Due to influence of the regularly-disturbed (in particularly ocean) surface the spacecraft (SC), under specific motion modes, executes forced vibrations in the pitching plane as well as in the area of the screen [1]. Of the interest, from the point of view of the dynamic effects appearance is the case of the coupled vibrations during the fly at the optional angle to the wave. Mechanical analogy of this dynamic process can be a vibration model of the mathematical pendulum with a suspension center wagging according to the desire law and with a length, which is the temporal function. It should be mentioned, that existing solutions, as a rule, are reduced to the solution of Mathieu-Hill equation on the conditions that the recuperate moment has non-linear (in particular cube) character [1, 2] and dimensionless amplitude of the parametric excitation is a small quantity. However, in real conditions of aircraft exploitation, value of this parameter can be not small, that can leads to significant lost quality of dynamic characteristics. It is necessary to note that analytical solution of the nonlinear problem order *m* is especially important for establishment of dynamic stability regions and verifying the parametric excitation frequency area, in which model becomes dynamic unstable.

This paper continues the discussions of the previous publications [1, 2, 5] and deals with the influence of dynamic absorber attached to spacecraft and periodic in time damping coefficient on an approximate analytical solution of the problem.

Nonlinear differential equation of the problem. Hybrid asymptotic solution. Differential equation system, describing system oscillation process «SC – dynamic absorber» is as follows [2]:

$$I_x \frac{d^2\gamma}{dt^2} + n\frac{d\gamma}{dt} + C_{11}(t)\gamma + C_{22}(t)\gamma^m + I_{x^0}\left(y + y^0\right) = 0, \quad . \tag{1}$$
$$I_{x^0}\left(y + y^0\right) + n_0\gamma^0(t) + C_1(t)y^0(t) = 0$$

With due regard to the second equation of the system (1) the main differential equation of tangage motion acquires the form

$$I_x\ddot{\gamma}(t) + n_0\dot{\gamma}(t) + C_{11}\gamma(t) + C_{22}\gamma^m(t) = F\left(\gamma^0, t\right), \tag{2}$$

where

$$F\left(\gamma^0, t\right) = n_0\gamma^0(t) + C^0\gamma^0(t), \tag{3}$$

$$n = \frac{1}{1 + \frac{y_0}{h}\cos(\omega t)}\left(n_1 + n_2\right) + n_3 + n_0 .$$

Here n_0 is a coefficient belonging to the dynamic absorber; n_1 – to lift wing; n_2 belongs to stabilizer; n_3 – to vertical fin.

Therefore, the problem with attached dynamic absorber reduces itself to the right part $F\left(\gamma^0, t\right)$ of initial equation (2).

In such a case calculation chart takes the following form [2] presented in Fig. 1, where the frame of axis y^0oz^0 is related to the dynamic absorber.

Equation (2) can be presented in the form:

$$\varepsilon^2\ddot{\gamma}(t) + B_1(t)\gamma(t) + \eta\gamma(\tau)^m = F\left(\gamma^0, t\right), \tag{4}$$

where ε and η are asymptotic parameters.

Note, that for particular case, when $m = 3$, differential equation (4) corresponds to Mathieu – equation. Here it will be discussed general case for the parameter nonlinearity m.

Using hybrid approach on the basis of perturbation technique and three-term approximation according to phase-integral method [4], an approximate analytical solution of equation problem (4) is:

$$\gamma(\tau) = \frac{Exp\left[-\frac{\nu(\tau)}{2}\right]}{B_1^{\frac{1}{4}}(\tau)}\Big[SinK(\tau)\left(c_1 + \bar{c}_1(\tau) + \eta\bar{d}_1(\tau)\right) + \tag{5}$$
$$+CosK(\tau)\left(c_2 + \bar{c}_2(\tau) + \eta\bar{d}_2(\tau)\right)\Big],$$

where first term in (5) corresponds to linear homogeneous solution, second term – to influence of right side of equation (4), third term – to non linearity influence.

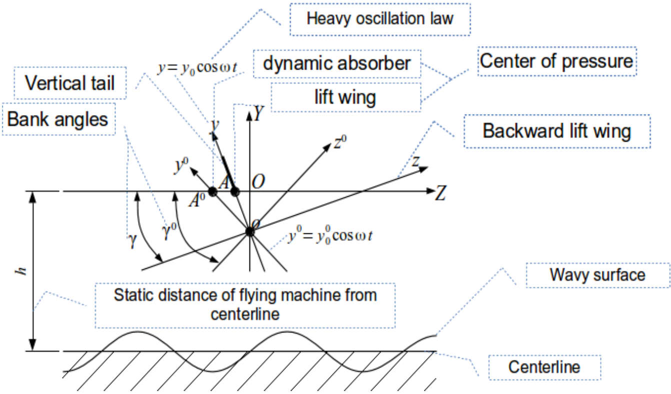

Fig. 1 – **Calculation chart «SC – absorber»**

Solution (5) can be rewrite as

$$\gamma(t) = Exp\left[-\frac{\nu(\tau)}{2}\right]\left\{SinK(\tau)\left[s_1 + \varepsilon\int \frac{F\left(\gamma^0, t\right) + \eta\left[\gamma_0(\tau)\right]^m}{B_1(\tau)^{0.25}} CosK(\tau)d\tau\right] + \right. \tag{6}$$

$$\left. + CosK(\tau)\left[s_2 - \varepsilon\int \frac{F\left(\gamma^0, t\right) + \eta\left[\gamma_0(\tau)\right]^m}{B_1(\tau)^{0.25}} SinK(\tau)d\tau\right]\right\},$$

where dimensionless parameters and solution functions are:

$$\gamma_0^0(\tau) = \frac{Exp\left[-\frac{\nu(\tau)}{2}\right]}{B_1^{\frac{1}{4}}(\tau)}\left[c_1 SinK(\tau) + c_2 CosK(\tau)\right],$$

$$t = \frac{2\tau}{\omega}, \quad \mu = \frac{a y_0}{h} = ka, \quad \varepsilon^2 = \frac{1}{a}, \tag{7}$$

$$a = \frac{4c_{11}}{I_x\omega^2}, \quad \nu = \frac{2n}{I_x\omega^2}, \quad \eta = \frac{4c_{22}}{I_x\omega^2}, \quad N(\tau) = \left[\gamma_0(\tau)\right]^m;$$

$$\bar{c}_1(t) = \int \frac{F\left(\gamma^0, t\right) y_2^0(t)}{-W(t)} dt, \quad \bar{c}_2(t) = -\int \frac{F\left(\gamma^0, t\right) y_1^0(t)}{-W(t)} dt; \tag{8}$$

$$\gamma_1^0(t) = \frac{Exp\left[-\frac{\nu(\tau)}{2}\right] SinK(\tau)}{B_1(\tau)^{0.25}}, \quad \gamma_2^0(t) = \frac{Exp\left[-\frac{\nu(\tau)}{2}\right] CosK(\tau)}{B_1(\tau)^{0.25}}; \quad (9)$$

$$W(\tau) = \frac{Exp\left[-\frac{\nu(\tau)}{2}\right] K'(\tau)}{-B_1^{0.5}(\tau)}; \quad (10)$$

$$\bar{c}_1(\tau) = \varepsilon \int \frac{F\left(\gamma^0, \tau\right) Exp\left[-\frac{\nu(\tau)}{2}\right] CosK(\tau)}{B_1^{0.5}(\tau)} d\tau,$$

$$\bar{c}_2(\tau) = -\varepsilon \int \frac{F\left(\gamma^0, \tau\right) Exp\left[-\frac{\nu(\tau)}{2}\right] SinK(\tau)}{B_1^{0.5}(\tau)} d\tau; \quad (11)$$

$$\bar{d}_1(\tau) = \varepsilon \int \frac{\eta N\left(\gamma^0, \tau\right) CosK(\tau)}{B_1^{0.5}(\tau)} d\tau,$$

$$\bar{d}_2(\tau) = -\varepsilon \int \frac{\eta N\left(\gamma^0, \tau\right) CosK(\tau)}{B_1^{0.5}(\tau)} d\tau; \quad (12)$$

$$K(\tau) = \int \varepsilon^{-1} B_1^{\frac{1}{2}}(\tau) d\tau; \quad (13)$$

$$B_1(\tau) = 1 - 2\frac{\mu}{a} Cos2\tau - \frac{\nu(\tau)^2}{4a} - \frac{\dot{\nu}(\tau)}{2a}; \quad (14)$$

$$c_{11}(\tau) = \frac{1}{\left(\frac{h}{y_0} + \cos\omega\tau\right) + \left(1 + \frac{y_0}{h}\cos\omega\tau\right)} \left(C_1 + C_2\right), \quad (15)$$

$$c_{22}(\tau) = \frac{1}{\left(\frac{h}{y_0} + \cos\omega\tau\right)\left(1 + \frac{y_0}{h}\cos\omega\tau\right)} \left(3C_1 \frac{l_1^2}{7h_1^2} + 3C_2 \frac{l_2^2}{7h_2^2}\right), \quad (16)$$

where C_1, C_2 – have to be obtained from [1]. c_1, c_2 – are constants, determined from the given initial conditions.

For the given parameters of spacecraft structure and initial conditions expression (5) is an approximate analytical solution problem as function of the amplitude of periodic in time damping coefficient *n* and parameter y_0/h (Fig. 2– Fig. 8).

Numerical examples.

1. Linear homogeneous problem:

$$B_1(\tau)=1-2\frac{\mu}{a}Cos2\tau-\frac{\nu(\tau)^2}{4a}-\frac{\dot{\nu}(\tau)}{2a},$$

$$\nu(\tau)=\nu_0(1+0.1\tau),\ a=100,\ \varepsilon=0.1,\ \nu_0=1,\ \frac{\mu}{a}=0.1.$$

For initial conditions

$$\gamma_0^0(0)=1,\quad \gamma_0'^0(\tau)=0$$

an one-term WKB approximation is:

$$\gamma_0^0(\tau)=Exp\left[-\frac{(1+0.1\tau)}{2}\right]\left\{0.0092Sin\left[8.94\tau+0.747\tau^3\right]+\right.$$

$$\left.+1.6487Cos\left[8.94\tau+0.747\tau^3\right]\right\}.$$

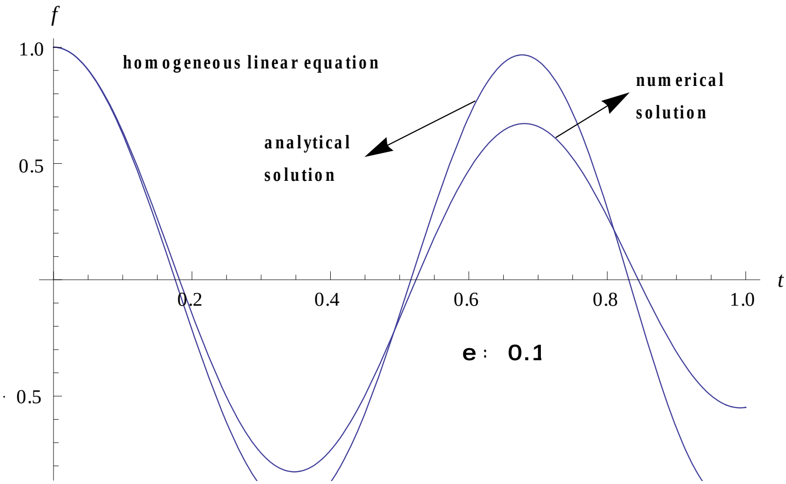

Fig. 2 – **Comparison of numerical and analytical solutions of linear homogeneous equation**

2. Linear nonhomogeneous equation:

$$\gamma_0(\tau)=Exp\left[-\frac{(1+0.1\tau)}{2}\right]\left\{\left(0.92\cdot10^{-2}+4\cdot10^{-4}\right)Sin\left[8.94\tau+0.747\tau^3\right]+\right.$$

$$\left.+(1.6487-0.052)Cos\left[8.94\tau+0.747\tau^3\right]\right\}.$$

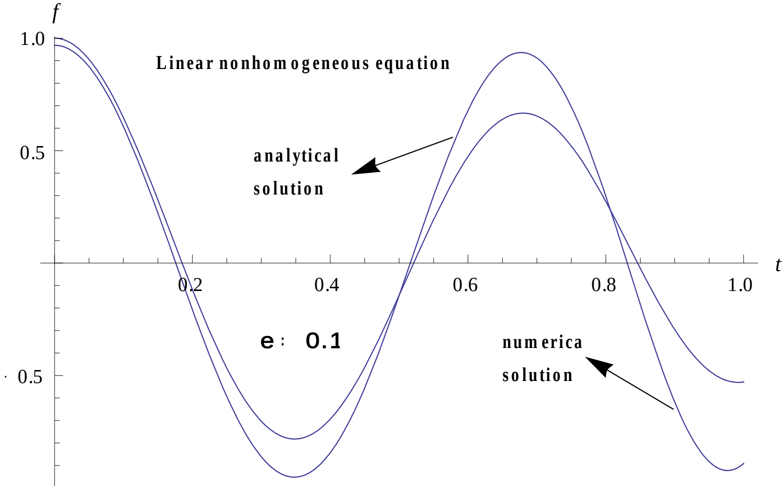

Fig. 3 – **Comparison of numerical and analytical solutions of linear nonhomogeneous equation**

3. Nonlinear non homogeneous equation:

$$\gamma(\tau) = Exp\left[-\frac{(1+0.1\tau)}{2}\right]\left\{\left(0.92\cdot 10^{-2} + 4\cdot 10^{-4}\right)Sin\left[8.94\tau + 0.747\tau^3\right] + \right.$$

$$\left. +(1.6487 - 0.052)Cos\left[8.94\tau + 0.747\tau^3\right]\right\}.$$

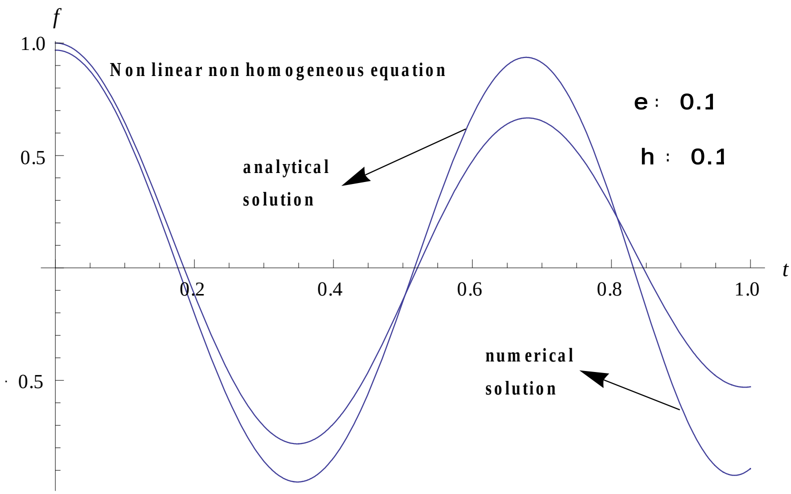

Fig. 4 – **Comparison of numerical and analytical solutions of nonlinear nonhomogeneous problem**

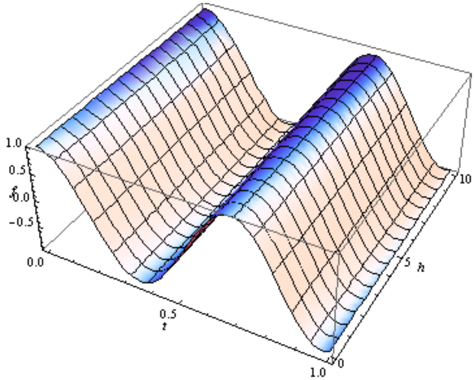

Fig. 5 – **Influence of nonlinear parameter** η

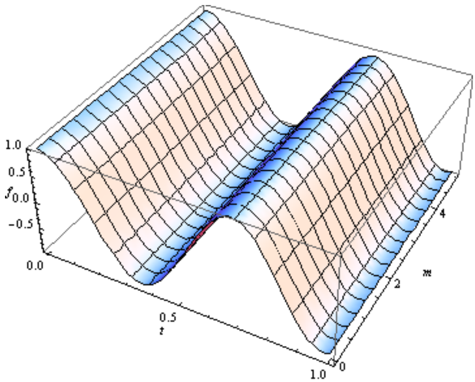

Fig. 6 – **Influence of nonlinear order function** m

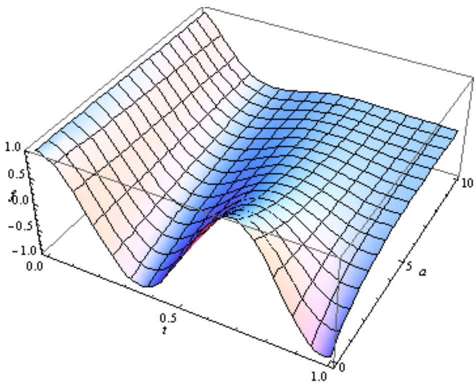

Fig. 7 – **Influence of damping function coefficient** α

$$\nu(\tau) = \nu_0 (1 + \alpha\tau)$$

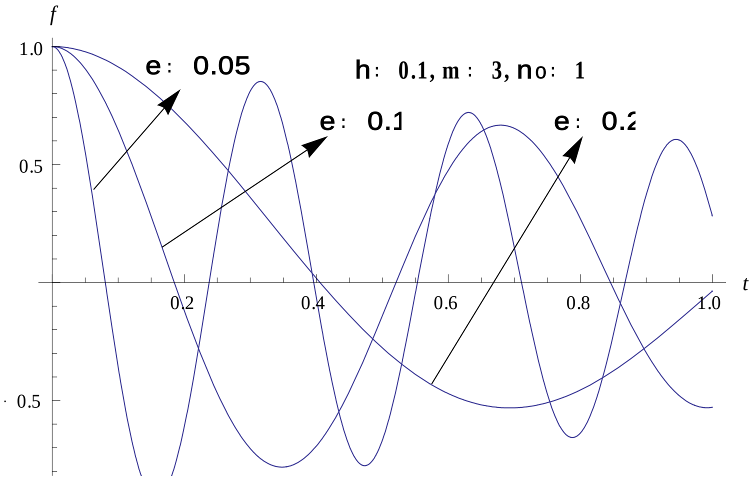

Fig. 8 – **Influence of singular perturbation parameter** ε

Conclusions. On the basis of hybrid asymptotic approach analytical solution for damping of nonlinear vibration problem of spacecraft structure with periodic damping coefficient, which moves near wavy surface, is suggested. Solution of the problem significantly depends on degree m of its nonlinearity and value of dimensionless disturbance range. On the certain time variation range behavior of studied system for the given nonlinear function is not too sensitive towards indicator m, that corresponds to [6]. For widening the range of oscillation of scalar parameters of asymptotic decomposition the usage of hybrid WKB-Galerkin method (interior asymptotic) on the basis of perturbation method (exterior asymptotic) is considered effective. Suggested hybrid asymptotic approach can be efficient in the cases when distance h of spacecraft from centerline of wave is the function of time (presence of variable coefficient at nonlinear component) and portability of damping coefficient n, which is perspective for further investigations of dynamic stability of structure moving near the wavy surface.

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КОЛЕБАНИЯ КОСМИЧЕСКОГО АППАРАТА С ПРИСОЕДИНЕННЫМ ДИНАМИЧЕСКИМ АБСОРБЕРОМ И ПЕРИОДИЧЕСКИМ КОЭФФИЦИЕНТОМ ДЕМПФИРОВАНИЯ ВБЛИЗИ ВОЗМУЩЕННОЙ ПОВЕРХНОСТИ

Обсуждается двухшаговый асимптотический подход на основе методов возмущений и фазовых интегралов (метод ВКБ) для приближенного аналитического решения проблемы нелинейных колебаний космического аппарата (с присоединенным динамическим абсорбером и периодическим во времени коэффициентом демпфирования), движущегося вблизи возмущенной поверхности. Решение сводится к необходимости интегрирования сингулярного нелинейного (порядка *m*) дифференциального уравнения с периодическими во времени коэффициентами при заданных начальных условиях. Полученное решение не ограничено безразмерной величиной параметрического возмущения и степенью нелинейности восстанавливающих сил и состоит из суммы двух функций в соответствии с методом возмущений (по скалярному параметру при нелинейной составляющей исходного уравнения) и ВКБ-приближения по параметру при старшей производной.

Ключевые слова: *динамика космического аппарата, волновая поверхность, присоединенный абсорбер, гибридный асимптотический подход, приближенное аналитическое решение.*

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КОЛИВАННЯ КОСМІЧНОГО АПАРАТА З ПРИЄДНАНИМ ДИНАМІЧНИМ АБСОРБЕРОМ ТА ПЕРІОДИЧНИМ КОЕФІЦІЄНТОМ ДЕМПФУВАННЯ ПОБЛИЗУ ЗБУРЕНОЇ ПОВЕРХНІ

Обговорюється двокроковий асимптотичний підхід на основі методів збурення і фазних інтегралів (метод ВКБ) для знаходження наближеного аналітичного розв’язку проблеми нелінійних коливань космічного апарату (з приєднаним динамічним абсорбером та періодичним за часом коефіцієнтом демпфування), який рухається поблизу збуреної поверхні. Розв’язування зводиться до інтегрування сингулярного нелінійного (порядку *m*) диференціального рівняння з періодичними за часом коефіцієнтами при даних початкових умовах. Здобутий розв’язок не обмежений безрозмірною величиною параметричного збурення та степенем нелінійності відновлювальних зусиль і складається із суми двох функцій у відповідності до методу збурення (за скалярним параметром при нелінійній складовій початкового рівняння) і ВКБ-наближення (за параметром при старшій похідній).

Ключові слова: *динаміка космічного апарату, хвильова поверхня, приєднаний абсорбер, гібридний асимптотический підхід, наближений аналітичний розв’язок.*

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