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OSCILLATIONS OF NONHOMOGENEOUS CYLINDRICAL SHELL STIFFENED BY CROSSED BARS WITH MEDIUM

In investigation of dynamic rigidity charcateristics of a nonhomogeneous cylindrical shell stiffened by crossed bars, the account of nonhomogeneity of material distribution and external medium effect was carried out by means of three-dimensional functional.

Keywords: *three-dimensional functional, nonhomogeneity, liquid medium, variation principle.*

Introduction. In the paper, eigen oscillations of a cylindrical shell stiffened by crossed nonhomogeneous bars in thickness i.e. of a solid medium system were studied.

For taking into account the nonhomogeneity in thickness of the cylindrical shell, two different methods may be used: by introducing sandwich [1] and nonhomogeneity function. In the paper, the nonhomogeneity was considered by accepting the Young's modulus and density of the material as a coordinate function changing in thickness.

Taking into account the nonhomogeneity in thickness of smooth cylindrical shells the parametric oscillations were studied in the papers [2–4]. Using the variation principle, for finding the oscillations frequency of the smooth cylindrical shell with medium, the frequency equation was constructed, its roots were found, and characteristic curves were constructed on force-frequency plane.

Problem statement. For taking into account the nonhomogeneity of the cylindrical shell in thickness we use a three-dimensional functional. In this case the total energy of the cylindrical shell is as follows:

$$\mathrm{U} = \frac{1}{2}\iint\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\alpha_{\alpha} e_{\alpha} + \sigma_{\beta} e_{\beta} + \tau_{\alpha\beta} e_{\alpha\beta} + \rho(z)\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)\right) d\alpha\, d\beta\, dz, \quad (1)$$

where

$$\sigma_\alpha = \frac{T_1}{h} + \frac{12M_1}{h^3}z; \sigma_\beta = \frac{T_2}{h} + \frac{12M_2}{h^3}z; \tau_{\alpha\beta} = \frac{s}{h} + \frac{12H}{h^3}z. \quad (2)$$

For taking into account nonhomogeneity, we accept the Young's modulus and the density of the material as a coordinate function changing in thickness [1]: $E = E(z)$ $\rho = \rho(z)$. It is assumed that the Poisson's ratio is constant. In this case the stress-strain relation is written as follows:

$$e_\alpha = \frac{1}{E(z)}\left(\sigma_\alpha - \nu\sigma_\beta\right);\ e_\beta = \frac{1}{E(z)}\left(\sigma_\beta - \nu\sigma_\alpha\right);\ e_{\alpha\beta} = \frac{2(1+\nu)}{E(z)}\sigma_{\alpha\beta}. \quad (3)$$

If we take into account expressions (2)–(3) and the equality

$$\iint \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\rho(z)\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right) d\alpha\, d\beta\, dz =$$

$$\iint \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\rho_0\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right) - 2\rho_1\left(\frac{\partial^2 w}{\partial x \partial t}\cdot\frac{\partial u}{\partial t} + \frac{\partial^2 w}{\partial y \partial t}\cdot\frac{\partial \vartheta}{\partial t}\right)^2 + \right.$$

$$\left. + \rho_2\left(\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 + \left(\frac{\partial^2 w}{\partial y \partial t}\right)^2\right) \right) d\alpha\, d\beta,$$

in (1), we can write :

$$V = \frac{1}{h}\iint \left\{ T_1\left[\frac{1}{E_0}\left(2T_2 - \nu T_1\right) + \frac{12}{E_1 h^3}\left(M_2 - \nu M_1\right)\right] + \right.$$

$$+ T_2\left[-\frac{\nu T_2}{E_0} + \frac{12}{E_1 h^3}\left(M_1 - \nu M_2\right)\right] + 2(1+\nu)\left(\frac{s}{E_0} + \frac{12H}{E_1 h^3}\right) + \quad (4)$$

$$\left. + \frac{72}{E_2 h^6}\left(2M_1 M_2 - \nu M_1^2 - \nu M_2^2 + 2(1+\nu)H^2\right) \right\} d\alpha\, d\beta.$$

The total energy of longitudinal bars and rings is:

$$V_1 = \frac{1}{2}\sum_{i=1}^{k_1}\int_{x_1}^{x_2}\left[E_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2 + E_iJ_{yi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + E_iJ_{zi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + G_iJ_{kpi}\left(\frac{\partial \varphi_{kpi}}{\partial x}\right)^2\right]dx +$$

$$+\sum_{i=1}^{k_1}\rho_iF_i\int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \frac{J_{kpi}}{F_i}\left(\frac{\partial \varphi_{kpi}}{\partial t}\right)^2\right]dx;$$

$$V_2 = \frac{1}{2}\sum_{j=1}^{k_2}\int_{y_1}^{y_2}\left[E_jF_j\left(\frac{\partial \vartheta_j}{\partial y} - \frac{w_j}{R}\right)^2 + E_jJ_{xi}\left(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2}\right)^2 + E_jJ_{zj}\left(\frac{\partial^2 u_i}{\partial y} - \frac{\varphi_{kpj}}{R}\right)^2\right] + \quad (5)$$

$$+\sum_{j=1}^{k_2}\rho_jF_j\int_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_j}{\partial t}\right)^2 + \left(\frac{\partial w_j}{\partial t}\right)^2 + \frac{J_{kpj}}{F_j}\left(\frac{\partial \varphi_{kpj}}{\partial t}\right)^2\right]dy.$$

The influence of medium on the cylindrical shell is presented by the external forces q_x, q_y, q_z. The work performed by these forces in location of the points of the shell is

$$A_0 = -\int_0^{x_1}\int_0^{2\pi}\left(q_x u + q_y \vartheta + q_z w\right)dxdy \; .$$

The total energy of the system under consideration is

$$W = V + V_1 + V_2 + A_0. \quad (6)$$

In expressions (1)–(6) u, ϑ, w are displacements of the shell, u_i, ϑ_i, w_i are the displacements of the points of longitudinal bar, u_j, ϑ_j, w_j are the displacements of the points of the ring, E_i is the modulus of elasiticity of the longitudinal bar, F_i is the area of cross-section of the longitudinal bar, G_i is the modulus of elasticity of the longitudinal bar at shear, $I_{yi}, I_{kp.i}$ are inertia moments of the cross section of the longitudinal bar, k_1 is the quantity of longitudinal bars, E, ν are the modulus of elasticity of the bar material and the Poisson's ratio, respectively, R, h are the radius and thickness of the cylindrical shell, respectively, E_j is the modulus of elasticity of the ring, F_j

is the square of the cross section of the ring, $I_{zj}, I_{zj}, I_{kp.j}$ are the inertia moments of the cross section of the ring, k_2 is the number of rings, q_x, q_y, q_z are the pressure force components influencing on the cylindrical shell from medium, and $\rho_i = \int_{-h}^{h.} \rho(z) z^i dz$, $\frac{1}{E_i} = \int_{-h}^{h.} \frac{z^i dz}{E(z)}$.

It is assumed that the following rigid contact conditions between the shell and rings are satisfied:

$$
\begin{gathered}
\mathrm{u_i}(x) = u(x, y_i) + h_i \varphi_1(x, y_i); \ \mathrm{v_i}(x) = v(x, y_i) + h_i \varphi_2(x, y_i); \\
\mathrm{w_i}(x) = w(x, y_i); \ \varphi_i(x) = \varphi_1(x, y_i); \ \varphi_{\kappa p \mathrm{i}}(x) = \varphi_2(x, y_i); \ h_i = 0{,}5h + H_i^1; \\
u_j(y) = u(x_j, y) + h_j \varphi_1(x_j, y); \ \vartheta_j(y) = \vartheta(x_j, y) + h_j \varphi_2(x_j, y); \qquad (7) \\
w_j(y) = w(x_j, y); \ \varphi_j(y) = \varphi_2(x_j, y); \ \varphi_j(y) = \varphi_2(x_j, y); \\
\varphi_{kpj}(y) = \varphi_1(x_j, y); \ h_j = 0{,}5h + H_j^1 .
\end{gathered}
$$

The system of motion equations of the medium in cylindrical coordinates, by means of the Lame system of equations is written as follows [5]:

$$
\begin{gathered}
(\lambda_s + 2\mu_s) \frac{\partial \theta}{\partial r} - \frac{2\mu_s}{r} \frac{\partial \omega_x}{\partial \varphi} + 2\mu_s \frac{\partial \omega_\varphi}{\partial x} - \rho_s \frac{\partial^2 s_x}{\partial t^2} = 0; \\
(\lambda_s + 2\mu_s) \frac{1}{r} \frac{\partial \theta}{\partial \varphi} - 2\mu_s \frac{\partial \omega_r}{\partial x} + 2\mu_s \frac{\partial \omega_x}{\partial x} - \rho_s \frac{\partial^2 s_\varphi}{\partial t^2} = 0; \qquad (8) \\
(\lambda_s + 2\mu_s) \frac{\partial \theta}{\partial x} - \frac{2\mu_s}{r} \frac{\partial}{\partial r}(r \omega_\varphi) + \frac{2\mu_s}{r} \frac{\partial \omega_r}{\partial \varphi} - \rho_s \frac{\partial^2 s_r}{\partial t^2} = 0.
\end{gathered}
$$

Here s_x, s_φ, s_r are the displacement vector components of the medium, λ_s, μ_s are the Lame coefficients, ρ_s is the density of the medium, x, r, φ are longitudinal, radial and circular coordinates respectively, and $a_t = \sqrt{\frac{\lambda_s + 2\mu_s}{\rho_s}}$, $a_e = \sqrt{\frac{\mu_s}{\rho_s}}$.

The volumetric extension θ and the components $\omega_x, \omega_\varphi, \omega_r$ are calculated by the following expressions:

$$\theta=\frac{\partial s_r}{\partial r}+\frac{s_r}{r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial \varphi}+\frac{\partial s_x}{\partial x};\ 2\omega_x=\frac{1}{r}\left[\frac{\partial\left(rs_\varphi\right)}{\partial r}-\frac{\partial s_r}{\partial \varphi r}\right];$$

$$2\omega_\varphi=\frac{\partial s_r}{\partial x}-\frac{\partial s_x}{\partial r};\ 2\omega_r=\frac{1}{r}\frac{\partial s_x}{\partial \varphi}-\frac{\partial s_\varphi}{\partial x}.$$

The stresses in the medium are expressed by the displacements s_x, s_φ, s_r as follows:

$$\sigma_{rx}=\mu_s\left(\frac{\partial s_x}{\partial r}+\frac{\partial s_r}{\partial x}\right);\ \ \sigma_{r\varphi}=\mu_s\left[r\frac{\partial}{\partial r}\left(\frac{s_\varphi}{r}\right)+\frac{1}{r}\frac{\partial s_r}{\partial \varphi}\right];$$

$$\sigma_{rr}=\lambda_s\left(\frac{\partial s_x}{\partial x}+\frac{1}{r}\frac{\partial\left(rs_r\right)}{\partial r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial \varphi}\right)+2\mu_s\frac{\partial s_r}{\partial r}. \tag{9}$$

Problem solution. When studying the oscillations of a visco-elastic cylindrical shell stiffened by bars with medium, we consider two cases: a) inertial influence of the medium on the oscillation process is weak; b) the inertial influence of the medium can not be ignored when investigating the oscillation prosess.

In the case a) the displacements of the medium will be as follows:

$$s_x=\left[\left(-kr\frac{\partial I_n\left(kr\right)}{\partial r}-4\left(1-\nu_s\right)kI_n\left(kr\right)\right)A_s+kI_n\left(kr\right)B_s\right]\cos n\varphi\cos kx\sin\omega t;$$

$$s_\theta=\left[-\frac{n}{r}I_n\left(kr\right)B_s-\frac{\partial I_n\left(kr\right)}{\partial r}C_s\right]\sin n\varphi\sin kx\sin\omega t;$$

$$s_r=\left[-k^2rI_n\left(kr\right)A_s+\frac{\partial I_n\left(kr\right)}{\partial r}B_s+\frac{n}{r}I_n\left(kr\right)C_s\right]\cos n\varphi\sin kx\sin\omega t. \tag{10}$$

In the case b) they are:

$$s_x=\left[A_skI_n\left(\gamma_e r\right)-\frac{C_s\gamma_t^2}{\mu_t}I_n\left(\gamma_t r\right)\right]\cos n\varphi\cos kx\sin\omega t;$$

$$s_\theta=\left[-\frac{A_s n}{r}I_n\left(\gamma_e r\right)-\frac{C_s nk}{r\mu_t}I_n\left(\gamma_t r\right)-\frac{B_s}{n}\frac{\partial I_n\left(\gamma_t r\right)}{\partial r}\right]\sin n\varphi\sin kx\sin\omega t; \tag{11}$$

$$s_r=\left[A_s\frac{\partial I_n\left(\gamma_e r\right)}{\partial r}-\frac{C_s k}{\mu_t}\frac{\partial I_n\left(\gamma_t r\right)}{\partial r}\frac{B_s n}{r}I_n\left(\gamma_t r\right)\right]\cos n\varphi\sin kx\sin\omega t.$$

The system of motion equations (8) was complemented with contact conditions. We will assume that the tangential surfaces of the cylindrical shell and medium can be changed with respect to each other and in the deformation process they are not separated from each other. In this case, in the sections $x = x_1$ and $x = x_2$ the conditions $\sigma_{xx} = 0;\ s_\theta = s_r = 0$ should be fulfilled.

The equality condition of normal components of displacements is:

$$s_r = w(r = R). \tag{12}$$

The equality conditions of pressure forces are:

$$q_x = 0,\ \ q_y = 0,\ \ q_z = -\sigma_{rr} \quad (r = R). \tag{13}$$

Show the expressions of pressure components q_z in the following way:

$$q_z = q_z^{(0)} C \cos n\varphi \sin kx \sin \omega t. \tag{14}$$

By means of contact conditions (9) and (10), and the system of motion equations of the medium, for $q_z^{(0)}$ we get the expression in the case a):

$$q_z^{(0)} - \mu_s \Delta^{-1} \Big\{ \Big(2(1-2\nu_s) I_n(k^*) + 2k^* I_n'(k^*) \Big) k^{*2} \times$$
$$\times \left[2k^{*2}\left(k^{*2} - n^2\right) \frac{I_n'(k^*)}{I_n(k^*)} + 2n^2 k^* \right] - 2\Big(k^* I_n'(k^*) - \left(k^{*2} + n^2\right) I_n(k^*) \Big) k^{*2} \times$$
$$\times \left[2(3-2\nu_s) k^* \frac{I_n'(k^*)}{I_n(k^*)} - 2n^2 k^* \right] + 2n\Big(I_n(k^*) - k^* I_n'(k^*) \Big) k^{*3} \times \tag{15}$$
$$\times \left[2(3-2\nu_s) k^* \frac{I_n'(k^*)}{I_n(k^*)} - 2n^2 \right] \Big\},$$

in the case b) we obtain:

$$q_z^{(0)} = \frac{E_s}{1+\nu_s} I_n(\gamma_l^*) \left[\frac{I_n(\gamma_t^*)}{I_n(\gamma_l^*)} \left(-\gamma_l^* \frac{I_n'(\gamma_t^*)}{I_n(\gamma_l^*)} + \gamma_l^{*2} + n^2 - \frac{\nu_s}{1-2\nu_s} \mu_l^{*2} \right) \times \right.$$

$$\times\frac{-n^2k^{*2}\mu_t^*+\dfrac{R^4k^{*3}\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_l^*\right)}}{-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_l^*\right)}}+\frac{2nk^*\gamma_l^*\mu_t^*+\dfrac{I_n^{'}\left(\gamma_l^*\right)}{I_n\left(\gamma_l^*\right)}+\dfrac{2nk^{*3}\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}}{-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_l^*\right)}}\times$$

$$\times\left(-n^2+n\gamma_t^*\frac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}+\frac{\nu_s}{1-2\nu_s}n\gamma_t^*\left(\gamma_t^*-\gamma_t^*\frac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}\right)\right)+$$

$$+\left(\frac{k^*\gamma_t^*}{\mu_t^*}\frac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}+\gamma_t^{*2}+\frac{\nu_s}{1-2\nu_s}\frac{2k^*\gamma_t^{*2}}{\mu_t^*}\right)\times \tag{16}$$

$$\left.\times\frac{\left[-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_l^*\right)}\right]}{2k^{*2}\gamma_l^*\gamma_t^*\dfrac{I_n^{'}\left(\gamma_l^*\right)}{I_n\left(\gamma_l^*\right)}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_l^*\right)}-2n^2k^{*2}}\right].$$

In the expressions (10), (11), (15), (16) $A_s,\ B_s,\ C_s$ are the unknown constants, $k,\ n,\ \gamma_e,\ \gamma_t$ are the wave numbers, I_n is the modified, *n* order, first kind Bessel function, $\gamma_e^2=k^2-\mu_e^2$, $\gamma_t^2=k^2-\mu_t^2$, $k^*=kR,\ \omega$ is the unknown frequency.

In the expression (6), the variational quantities are $u,\vartheta,w.T_1,T_2,M_1,M_2,S,H$. Determine the stationary value of the functional (6). For that we use the Ritz method. We will look for the unknown quantities in the form:

$$u=\cos\frac{\pi x}{l}\sin\left(k\varphi\right)\left(u_0\cos\omega t+u_1\sin\omega t\right);$$

$$\vartheta=\sin\frac{\pi x}{l}\cos\left(k\varphi\right)\left(\vartheta_0\cos\omega t+\vartheta_1\sin\omega t\right);$$

$$w=\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(w_0\cos\omega t+w_1\sin\omega t\right);$$

$$T_1=\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(T_{10}\cos\omega t+T_{11}\sin\omega t\right);$$

$$T_2 = \cos\frac{\pi x}{l}\cos(k\varphi)(T_{20}\cos\omega t + T_{21}\sin\omega t);$$

$$S = \sin\frac{\pi x}{l}\sin(k\varphi)(S_{10}\cos\omega t + S_{11}\sin\omega t);$$

$$M_1 = \sin\frac{\pi x}{l}\sin(k\varphi)(M_{10}\cos\omega t + M_{11}\sin\omega t); \quad (17)$$

$$M_2 = \cos\frac{\pi x}{l}\sin(k\varphi)(M_{20}\cos\omega t + M_{21}\sin\omega t);$$

$$H = \cos\frac{\pi x}{l}\cos(k\varphi)(H_{10}\cos\omega t + H_{11}\sin\omega t).$$

If we substitute the expressions (12) in the functional (6), we get a function dependent on the variables $u_0, u_1, \vartheta_0, \vartheta_1, w_0, w_1$, $T_0, T_1, T_{20}, T_{21}, S_{10}, S_{11}$, $M_{10}, M_{12}, M_{20}, M_{22}, H_{10}, H_{11}$. The stationarity condition of the obtained function is determined from the following system:

$$1)\frac{\partial J}{\partial u_0} = 0;\ \ 2)\frac{\partial J}{\partial u_1} = 0;\ 3)\frac{\partial J}{\partial \vartheta_0} = 0;\ \ 4)\frac{\partial J}{\partial \vartheta_1} = 0;$$

$$5)\frac{\partial J}{\partial w_0} = 0;\ \ 6)\frac{\partial J}{\partial w_1} = 0; 7)\frac{\partial J}{\partial T_0} = 0;\ \ 8)\frac{\partial J}{\partial T_{11}} = 0; \quad (18)$$

$$9)\frac{\partial J}{\partial T_{20}} = 0;\ \ 10)\frac{\partial J}{\partial T_{21}} = 0; 11)\frac{\partial J}{\partial S_{10}} = 0;\ \ 12)\frac{\partial J}{\partial S_{11}} = 0.$$

As the system (18) is homogeneous, for the existence of its nontrivial solution, the principal determinant should be equal to zero. As a result, we get the following frequency equation:

$$\det\left\|a_{ij}\right\| = 0 \quad i, j = 1,18. \quad (19)$$

Equation (19) was investigated by numerical method. For the parameters of the medium and shell, the following values were taken:

$$h^* = \frac{h}{R} = 0,25\cdot 10^{-2};\ \nu = 0,3;\ E_j = E = E_i = 6,67\cdot 10^9 H/m^2;\ \alpha = 0,5;$$

$$\rho_j = 0,26\cdot 10^{-2} N\cdot sm^2/m^2;\ a = 2,25a_t;\ a_t = 308\, m/sm;$$

$$E_0 = E; \rho_0 = \rho_j, h_j = 1,39mm;\ F_j = 5,75mm^2;$$

$$J_{xj} = 19,9mm^4; \frac{J_{zj}}{2\pi R^3 h} = 0,23\cdot 30^{-6}; J_{kp.j} = 0,48mm^4.$$

$$\frac{F_j}{2\pi Rh} = 0,1591\cdot 10^{-1};\ \nu = 0,3;\ \frac{I_{yi}}{2\pi R^3 h} = 0,8289\cdot 10^{-6};$$

$$\rho_i = 0,26 \cdot 10^4 N \cdot sm^2 / m^2; \ \frac{I_{zi}}{2\pi R^3 h} = 0,1326 \cdot 10^{-6};$$

$$\frac{I_{kpi}}{2\pi R^3 h} = 0,5305 \cdot 10^{-6}; \ h_i = 0,01375R; \ G_i = \frac{E_i}{2(1+\nu)}.$$

We considered two cases of the change of the nonhomogeneity function: linear $E(z) = E_0\left[1+\alpha\left(\frac{z}{h}\right)\right]$, $\rho(z) = \rho_0\left[1+\alpha\left(\frac{z}{h}\right)\right]$ and parabolic $E(z) = E_0\left[1+\alpha\left(\frac{z}{h}\right)^2\right]$, $\rho(z) = \rho_0\left[1+\alpha\left(\frac{z}{h}\right)^2\right]$. Here the Young's modulus, α is the nonhomogeneity parameter. Note that in the case of linear function $|\alpha| < 1$, in the parabolic principle case α is any number, and $\omega_1 = \sqrt{(1-\nu^2)\rho_0 R^2 \omega^2 / E}$.

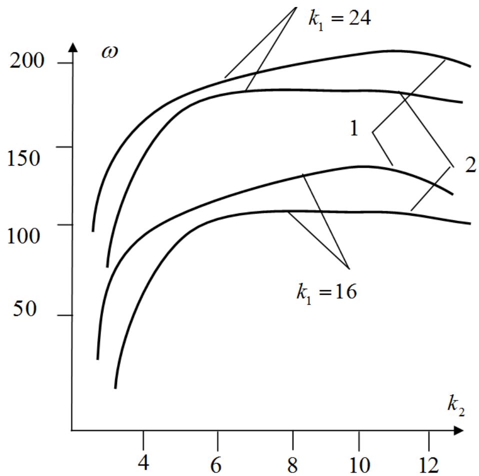

Fig. 1 – **Dependence of the frequency parameter on the amount of rings: 1 – the linear law; 2 – the parabolic law**

The results of calculations were given in Fig. 1 for different k_1 in the form of dependence of the system oscillation frequency on the amount of rings. The lines of nonhomogeneity curves correspond to curves 1, parabolic change cases of nonhomogeneity laws to curves 2.

Conclusions. The calculations show that the oscillation frequencies corresponding to linear case of nonhomogeneity laws are greater than the oscillation frequencies corresponding to the case of parabolic change. As it is seen from the Fig. 1, by increasing the number of rings, at first the oscillation frequencies of the system increase and after some number of rings decreases. This is explained by the fact that by increasing the number of rings, their mass increases and inertial effect on the oscillation process strengthens. Furthermore, the oscillation frequencies of the system increases due to increase of the number of longitudinal bars.

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КОЛИВАННЯ НЕОДНОРІДНОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ, ПОСИЛЕНОЇ ПЕРЕХРЕСНИМИ СИСТЕМАМИ РЕБЕР, СУМІСНІ З СЕРЕДОВИЩЕМ

Проведено дослідження динамічних жорсткісних характеристик неоднорідних циліндричних оболонок, посилених перехресними системами ребер, використовуючи тривимірний функціонал з урахуванням неоднорідності розподілу матеріалу та впливу зовнішнього середовища.

Ключові слова: *тривимірний функціонал, неоднорідність, рідке середовище, варіаційний принцип.*

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КОЛЕБАНИЯ НЕОДНОРОДНОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ, ПОДКРЕПЛЕННОЙ ПЕРЕКРЕСТНЫМИ СИСТЕМАМИ РЁБЕР, СОВМЕСТНЫЕ СО СРЕДОЙ

Проведено исследование динамических жёсткостных характеристик неоднородных цилиндрических оболочек, усиленных перекрёстными системами рёбер, используя трёхмерный функционал с учётом неоднородности распределения материала и влияния внешней среды.

***Ключевые слова*:** *трехмерный функционал, неоднородность, жидкая среда, вариационный принцип.*

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