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OSCILLATION OF ANISOTROPIC INHOMOGENEOUS CYLINDRICAL SHELL WITH LIQUID SUPPORTED BY CROSS-RIBS

The free oscillation of an anisotropic inhomogeneous rib of a cylindrical shell supported by cross-systems of edges and in contact with a moving fluid is investigated. Using the Hamilton – Ostrogradsky variational principle, a system of equations of motion supported by cross systems of ribs, non-uniform in thickness by an anisotropic cylindrical shell in contact with a moving fluid, was constructed. To take into account the heterogeneity of the shell material in thickness, it is assumed that the Young's modulus and density of the shell material are functions of the normal coordinate. Frequency equations are constructed and implemented numerically. Characteristic curves of dependence are built.

Keywords: *reinforced shell, variational principle, fluid, free oscillation, anisotropic shell.*

Introduction. When designing modern devices, machines and structures, an important role is played by the calculations for the stability, oscillation and strength of thin-walled elements of the shell-type structures in contact with the medium. Such structures can be in contact with a liquid and be subjected not only to static loads, but also to dynamic ones. To give greater rigidity of the shell are supported by various ribs. However, the behavior of inhomogeneous anisotropic thin-walled structural elements with ribs, taking into account their discrete location, the influence of the fluid has not been adequately investigated. Therefore, the development of mathematical models for studying the behavior of reinforced inhomogeneous anisotropic shells that most fully take into account their work under dynamic loads, and conducting studies of stability and oscillations based on them, as well as the selection of rational parameters of structures in contact with a liquid, are urgent tasks.

We note that the study of free vibrations of ribbed isotropic homogeneous cylindrical shells filled with flowing fluid is described in [1, 9]. The effects of the number of ribs, their rigidity, fluid flow velocity, various mechanical, physical, and geometric dimensions of the shell on the natural vibration frequencies and the optimization parameter of a circular ribbed cylindrical shell are studied. The works [4, 5] are devoted to the study of the free oscillation of an isotropic inhomogeneous rib of a cylindrical shell supported by cross systems of edges in contact with a moving fluid. Using the Hamilton – Ostrogradsky variational principle, systems of equations of motion supported by cross systems of ribs, non-uniform in thickness by an anisotropic cylindrical shell in contact with a moving fluid, are constructed. The results of an experimental study of the effect of reinforcing ribs and attached solids on the frequencies and forms of free vibrations of thin elastic structurally inhomogeneous shells are presented in [3]. In the papers [6, 7], using the asymptotic method, the frequency equations of smooth cylindrical shells filled with liquid were constructed, approximate frequencies of the equation and simple calculation formulas were obtained, which allow finding the values of the minimum natural frequencies of the oscillations of the considered system. Here, the forced vibrations of a reinforced shell filled with a liquid are investigated, and the amplitude - frequency characteristics of the considered oscillatory processes are determined. The works [10 – 12] are devoted to the study of parametric oscillations of a non-linear and non-uniform in thickness rectilinear rod in a visco-elastic medium with the use of the Pasternak contact model. The effects of the main factors - the elasticity of the base, the damage of the material of the rod and the shell, the dependence of the shear factor on the oscillation frequency on the characteristics of the longitudinal vibrations of the points of the rod in a visco-elastic medium are studied. In all the cases studied, the dependences of the zone of dynamic stability of vibrations of a rod in a visco-elastic medium on the parameters of the structure on the load-frequency plane are plotted.

Formulation of the problem. Anisotropic inhomogeneous ribbed shell is considered as a system consisting of its own shell and rigidly connected edges along the lines of contact. It is assumed that the stress-strain state of the shell can be completely determined in the framework of the linear theory of elastic thin shells, based on the Kirchhoff – Love hypotheses, and the theory of Kirchhoff – Clebsh curvilinear rods is applicable to the calculation of the ribs. The coordinate system is chosen so that the coordinate lines coincide with the lines of the main curvatures of the middle surface of the shell. It is assumed that the edges are placed along the coordinate lines, and their edges, like the edges of the shell, lie in the same coordinate plane. In addition, it is assumed that all edges form a regular system. A regular system of longitudinal and annular ribs is understood to be such a system in which the stiffness of all the ribs, their mutual distances are equal, and the distance from the edge of the shell to the nearest edge to it is equal to the distance between the ribs.

The deformed state of the shell can be determined through three components of the displacements of its middle surface u, ϑ and w. In this case, the angles of rotation of normal elements φ_1, φ_2 relative to the coordinate lines y and x are expressed through w and ϑ with the help of dependencies

$$\varphi_1 = -\frac{\partial w}{\partial x}, \qquad \varphi_2 = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right),$$

where $R-$ the radius of the middle surface of the shell.

To describe the deformed state of the ribs, in addition to the three components of the displacements of the centers of gravity of their cross sections (u_j, ϑ_j, w_j first transverse rod u_i, ϑ_i, w_i of the longitudinal rod), it is also necessary to determine the twist angles φ_{kpi} and φ_{kpj}.

Taking into account that, according to the accepted hypotheses, there is a constancy of radial deflections along the height of the sections, as well as the equality of the corresponding twist angles resulting from the rigid connection of the ribs to the shell, we write the following relations:

$$
\begin{aligned}
&u_i(x) = u(x, y_i) + h_i\varphi_1(x, y_i); && \vartheta_i(x) = \vartheta(x, y_i) + h_i\varphi_2(x, y_i);\\
&w_i(x) = w(x, y_i); && \varphi_i = \varphi_1(x, y_i);\\
&\varphi_{kpi}(x) = \varphi_2(x, y_i); && u_j(y) = u(x_j, y) + h_j\varphi_1(x_j, y);\\
&\vartheta_j(x) = \vartheta(x_j, y) + h_j\varphi_2(x_j, y); && w_j(x) = w(x_j, y);\\
&\varphi_j = \varphi_2(x_j, y); && \varphi_{kpj}(x) = \varphi_1(x_j, y).
\end{aligned}
$$

Here $h_i = 0{,}5h + H_i^1$, $h_j = 0{,}5h + H_j^1$, $h -$ shell thickness, H_i^1 and H_j^1 – distance from i -th longitudial and j -th cross rib to shell surface, x_i and y_i – interface line coordinates of ribs with shell, φ_i, φ_{kpi} and φ_j, φ_{kpj} – angels of bending and twisting of cross-section of longitudial and transverse ribs respectivel.

Regarding external influences, it is assumed that the surface loads acting on the ribbed shell from the liquid can be reduced to the components q_x, q_y and q_z, applied to the middle surface of the shell.

Differential equations of motion and natural boundary conditions for a transversely supported orthotropic cylindrical shell with a fluid are derived from the Ostrogradsky – Hamilton variational principle. To do this, we first write down the potential and kinetic energy of the system.

To take into account the heterogeneity in thickness of the cylindrical shell, we will proceed from the three-dimensional functional. In this case, the total energy functional of a cylindrical shell has the form

$$V = \frac{1}{2}\iint\int_{-h/2}^{h/2}\left(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \rho(z)\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)dxdydz \quad (1)$$

There are various ways to account for the heterogeneity of the shell material. One of them is that the Young's modulus and the density of the shell material are accepted as functions of the normal coordinate z: $E = E(z)$, $\rho = \rho(z)$ [8]. It is assumed that the Poisson's ratio is constant. In this case, the ratio of strain-stress is:

$$\sigma_{11} = b_{11}(z)\varepsilon_{11} + b_{12}(z)\varepsilon_{22}; \quad \sigma_{22} = b_{12}(z)\varepsilon_{11} + b_{22}(z)\,\varepsilon_{22}; \quad \sigma_{12} = b_{66}(z)\varepsilon_{12}; \quad (2)$$

$$\varepsilon_{11}=\frac{\partial u}{\partial x};\qquad \varepsilon_{22}=\frac{\partial \vartheta}{\partial y}+w;\qquad \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x}. \tag{3}$$

Taking into account (2), (3) and

$$\iint\int_{-h/2}^{h/2}\left(\rho(z)\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dxdydz=$$

$$=\iint\left(\rho_0\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)-2\rho_1\left(\frac{\partial^2 w}{\partial x\partial t}\cdot\frac{\partial u}{\partial t}+\frac{\partial^2 w}{\partial y\partial t}\cdot\frac{\partial \vartheta}{\partial t}\right)+\right.$$

$$\left.+\rho_2\left(\left(\frac{\partial^2 w}{\partial x\partial t}\right)^2+\left(\frac{\partial^2 w}{\partial y\partial t}\right)^2\right)\right)dxdy$$

in (1), it can be written down

$$V=\frac{1}{2}\iint\left\{\tilde{b}_{11}\varepsilon_{11}^2+2\tilde{b}_{12}\varepsilon_{11}\varepsilon_{22}+2\tilde{b}_{26}\varepsilon_{12}\varepsilon_{22}+2\tilde{b}_{16}\varepsilon_{11}\varepsilon_{12}+\right.$$

$$\left.+\tilde{b}_{22}\varepsilon_{22}^2+\tilde{b}_{66}\varepsilon_{12}^2\right\}dxdy+\iint\left(\tilde{\rho}\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dxdy, \tag{4}$$

here $\tilde{b}_{11}=\int_{-h/2}^{h/2} b_{11}(z)dz;\qquad \tilde{b}_{12}=\int_{-h/2}^{h/2} b_{12}(z)dz;\qquad \tilde{b}_{22}=\int_{-h/2}^{h/2} b_{22}(z)dz;$

$\tilde{b}_{66}=\int_{-h/2}^{h/2} b_{66}(z)dz;\qquad b_{11}(z)=\dfrac{E_1(z)}{1-\nu_1\nu_2};\qquad b_{12}(z)=\dfrac{\nu_2E_1(z)}{1-\nu_1\nu_2}=\dfrac{\nu_1E_2(z)}{1-\nu_1\nu_2};$

$b_{22}(z)=\dfrac{E_2(z)}{1-\nu_1\nu_2};\qquad b_{66}(z)=G_{12}(z)=\mathrm{G}(z)-$ main elastic modules of orthotrop material, $\tilde{\rho}=\int_{-h}^{h.}\rho(z)dz$.

The expressions for the potential energy of the elastic deformation of the longitudinal and th transverse ribs are as follows [9]:

$$\Pi_i=\frac{1}{2}\int_0^L\left[\tilde{E}_iF_i\left(\frac{\partial u_i}{\partial x}\right)^2+\tilde{E}_iJ_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+\tilde{E}_iJ_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2+\tilde{G}_iJ_{\kappa pi}\left(\frac{\partial \varphi_{\kappa pi}}{\partial x}\right)^2\right]dx;$$

$$\Pi_j = \frac{1}{2}\int_0^{2\pi}\left[\tilde{E}_j F_j\left(\frac{\partial\vartheta_j}{\partial y} - \frac{w_j}{R}\right)^2 + \tilde{E}_j J_{xj}\left(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2}\right)^2 + \right.$$
$$\left. + \tilde{E}_j J_{zj}\left(\frac{\partial^2 u_i}{\partial y^2} - \frac{\varphi_{\kappa pj}}{R}\right)^2 + \tilde{G}_j J_{\kappa pj}\left(\frac{\partial\varphi_{\kappa pi}}{\partial y} + \frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\right]dy. \tag{5}$$

The kinetic energies of the ribs are written as [2]:

$$K_i = \rho_i F_i \int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial\vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \frac{J_{\kappa pi}}{F_i}\left(\frac{\partial\varphi_{\kappa pi}}{\partial t}\right)^2\right]dx;$$
$$K_j = \rho_j F_j \int_{y_1}^{y_2}\left[\left(\frac{\partial u_j}{\partial t}\right)^2 + \left(\frac{\partial\vartheta_j}{\partial t}\right)^2 + \left(\frac{\partial w_j}{\partial t}\right)^2 + \frac{J_{\kappa pj}}{F_j}\left(\frac{\partial\varphi_{\kappa pj}}{\partial t}\right)^2\right]dx. \tag{6}$$

In expressions (4) and (6), $F_i, J_{zi}, J_{yi}, J_{kpi}$ – the area and moments of inertia of the cross-section of the longitudinal rod, respectively, about an axis Oz and an axis Oy parallel to the axis and passing through the center of gravity of the section, as well as $F_j, J_{zj}, J_{yj}, J_{kpj}$ – its moment of inertia during torsion; similar values for the transverse rod; and $\tilde{E}_i, \tilde{G}_i$ – the modules of elasticity and shear $\tilde{E}_j, \tilde{G}_j$ of the material of the longitudinal and transverse rods, respectively, ρ_i, ρ_j – of the density of the materials from which the longitudinal and transverse rods are made.

The potential energy of external surface loads acting from an ideal fluid applied to the shell is defined as the work performed by these loads when the system is transferred from a deformed state to an initial non-deformed state and is represented as

$$A_0 = -\int_0^L\int_0^{2\pi} q_z w\,dx\,dy. \tag{7}$$

The total energy of the system is equal to the sum of the energies of the elastic deformations of the shell and transverse ribs, as well as the potential energies of all external loads acting from an ideal fluid

$$J = V + \sum_{j=1}^{k_2}\left(\Pi_j + K_j\right) + \sum_{i=1}^{k_1}\left(\Pi_i + K_i\right) + A_0. \tag{8}$$

here k_1 – is the number of longitudinal, k_2 – is the number of transverse ribs.

Assuming that the main flow velocity is equal U and the deviations from this velocity are small, let us use the wave equation for the potential φ of perturbed velocities along [13]

$$\Delta\tilde{\varphi}-\frac{1}{a_0^2}\left(\frac{\partial^2\tilde{\varphi}}{\partial t^2}+2U\frac{\partial^2\tilde{\varphi}}{R\partial\xi\partial t}+U^2\frac{\partial^2\tilde{\varphi}}{R^2\partial\xi^2}\right)=0. \tag{9}$$

The expression of the total energy of the system (8) and the equation for the motion of a fluid (9) are complemented by contact conditions. On the contact surface of the shell - liquid, the continuity of radial velocities and pressures is observed. The condition of impermeability or smoothness of flow around the shell wall is [13]

$$\vartheta_r\big|_{r=R}=\frac{\partial\varphi}{\partial r}\bigg|_{r=R}=-\left(\omega_0\frac{\partial w}{\partial t_1}+U\frac{\partial w}{R\partial\xi}\right). \tag{10}$$

Equality of radial pressure from the liquid to the shell:

$$q_z=-p_{|r=R}. \tag{11}$$

If in (9) and (10) we substitute $U=0$, then we obtain the equation of motion and the condition of impermeability or smoothness of flow around the shell wall for a fluid at rest. The frequency equation of a ribbed inhomogeneous orthotropic shell with a flowing fluid is obtained on the basis of the principle of stationarity of the Ostrogradsky – Hamilton action

$$\delta W=0, \tag{12}$$

where $W=\int_{t'}^{t''}Jdt$ is the Hamilton action, t' and t'' are given arbitrary points in time.

Complementing the contact conditions of the total energy of system (8), the equations of motion of a liquid (9) arrive at the problem of natural oscillations supported by a cross-system of ribs of a thickness that is non-uniform in thickness of an orthotropic cylindrical shell with a flowing fluid. In other words, the problem of natural oscillations, supported by a cross-system of edges, of an inhomogeneous orthotropic cylindrical shell with a flowing fluid, reduces to the simultaneous integration of expressions for the total energy of system (8), the equation for fluid motion (9) when conditions (9) and (10) are satisfied their contact.

Problem solving. The potential of perturbed speeds φ are looking for in the form

$$\tilde{\varphi}(\xi,r,\theta,t_1)=f(r)\cos n\varphi\sin\chi\xi\sin\omega_1 t_1. \tag{13}$$

In expression (8), the variable quantities are u, ϑ, w,. These unknown values are approximated as follows:

$$
\begin{aligned}
&u=u_0\cos\frac{\pi x}{l}\sin k\varphi\sin\omega t;\\
&\vartheta=\vartheta_0\sin\frac{\pi x}{l}\cos k\varphi\sin\omega t; \qquad (14)\\
&w=w_0\sin\frac{\pi x}{l}\sin k\varphi\sin\omega t.
\end{aligned}
$$

Substituting (14) into (8) after integration, we obtain a function of the variables $u_0,\ \vartheta_0,\ w_0$. The stationary value of the function obtained is determined by the following system:

$$1)\ \frac{\partial J}{\partial u_0}=0;\quad 2)\ \frac{\partial J}{\partial \vartheta_0}=0;\quad 3)\ \frac{\partial J}{\partial w_0}=0. \qquad (15)$$

Nontrivial solution of the system of linear algebraic equations (15) of the third order is possible only in the case when $\omega_1 -$ the root of its determinant. The definition ω_1 reduces to a transcendental equation, since ω_1 it is included in the arguments of the Bessel function

$$deta_{ij}=0,\quad i,j=1,3\,. \qquad (16)$$

Numerical results. The frequency equation (16) is solved numerically.

$$R=160\,мм;\ \ L=800\,мм;\ \ h=0,45мм;\ \ \tilde{E}_j=\tilde{E}_i=6,67\cdot10^9\,Па;$$

$$a_0=1430\,м/сек;\ \ \rho_j=\rho_i=7,8\ г/см^3;\ \ F_j=5,75\,мм^2;\ \ I_{xj}=19,9\,мм^4;$$

$$I_{kp.j}=0,48\,мм^4;\quad \nu_1=0,11;\quad \nu_2=0,19;\quad \frac{I_{kp.i}}{2\pi R^3h}=0,5305\cdot10^{-6};$$

$$\frac{I_{yi}}{2\pi R^3h}=0,8289\cdot10^{-6};\quad \frac{F_i}{2\pi Rh}=0,1591\cdot10^{-1};\quad \frac{J_{zi}}{2\pi R^3h}=0,13\cdot10^{-6}\,.$$

Two types of laws of variation of heterogeneity are considered

- linear $E_1(z)=E_1(1+\alpha z/h);\ E_2(z)=E_2(1+\beta z/h);\ \rho(z)=\rho_0(1+\alpha z/h)$;
- parabolic $E_1(z)=E_1\left(1+\alpha(z/h)^2\right);\ E_2(z)=E_2\left(1+\beta(z/h)^2\right);$

  $\rho(z)=\rho_0\left(1+\alpha(z/h)^2\right).$

Here, α,β are the heterogeneity parameters. Note, that with a linear law change, $|\alpha|<1,\ |\beta|<1$, with a parabolic change, α,β – are arbitrary. The counting results are shown in Fig. 1 and Fig. 2.

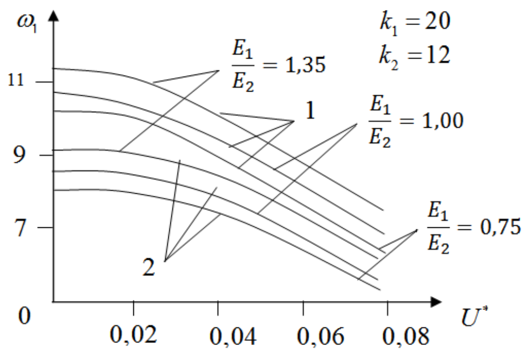

Fig.1 – **Зависимость параметра частот колебаний от скорости жидкости: 1 – линейный закон, 2 – параболический закон**

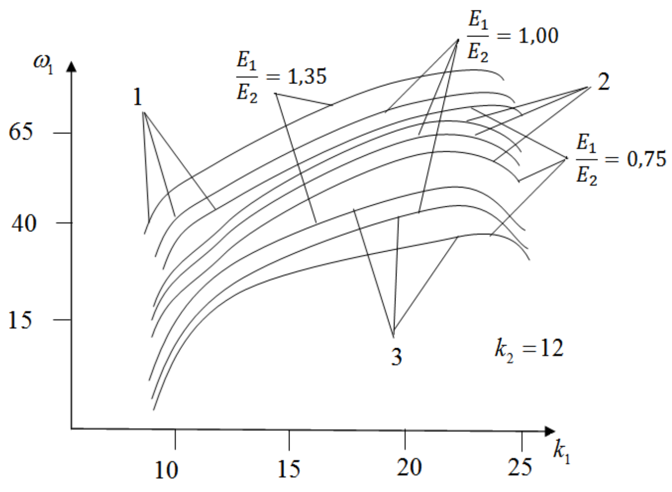

Fig. 2 – **The dependence of the frequency parameter on the number of longitudinal edges: 1 – uniform shell, 2 – linear law, 3 – parabolic law**

In Fig. 1 shows the dependences of the frequency parameter ω_1 on the relative flow velocity U^* for various laws of variation in heterogeneity across the shell thickness, and for different ratios E_1/E_2. It is seen that the increase in speed leads to a decrease in frequency. Note, that $U^* = 0$ corresponds to the resting fluid.

In Fig. 2 illustrates the effect of the number k_1 of longitudinal ribs on the parameters of the oscillation ω_1 frequency of the considered system. It is seen that with k_1 an increase in the parameters of the frequency of oscillation ω_1 of the system first increase, and then at a certain value k_1 they begin to decrease. This is because, with an increase k_1 in the weight of the rods, it increases and leads to a significant influence of their inertial properties on the oscillation process. Comparisons of the above graphs show that taking into account inhomogeneity leads to a decrease in the values of the natural frequencies of the considered system compared to the natural frequencies of the same system when the shell is homogeneous. In addition, with a decrease in the ratio E_1 / E_2 of the oscillation frequencies of the considered system, it also decreases compared to the natural frequencies of oscillations of the same system when the shell is isotropic.

Conclusions. The free oscillation of an anisotropic inhomogeneous rib of a cylindrical shell supported by cross-systems of edges and in contact with a moving fluid is investigated. Using the Hamilton – Ostrogradsky variational principle, a system of equations of motion supported by cross systems of ribs, non-uniform in thickness by an anisotropic cylindrical shell in contact with a moving fluid, was constructed. To take into account the heterogeneity of the shell material in thickness, it is assumed that the Young's modulus and density of the shell material are functions of the normal coordinate. In the study of free oscillations, supported by cross systems of ribs, anisotropic cylindrical shell inhomogeneous in thickness, in contact with a moving fluid, two cases are considered: a) the fluid inside the shell is at rest; b) the fluid inside the shell moves at a constant speed. In both cases, frequency equations are constructed and implemented numerically. In the process of calculations, linear and parabolic laws are adopted for the inhomogeneity function. Characteristic curves of dependence are built.

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КОЛИВАННЯ АНІЗОТРОПНОЇ НЕОДНОРІДНОЇ ПІДКРІПЛЕНОЇ ПЕРЕХРЕСНИМИ РЕБРАМИ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ З РІДИНОЮ

Досліджено вільні коливання анізотропної неоднорідної підкріпленої перехресними системами ребер циліндричної оболонки, що контактує з рухомою рідиною. Використовуючи варіаційний принцип Гамільтона – Остроградського отримані системи рівнянь її руху. З метою врахування неоднорідності матеріалу оболонки за товщиною приймається, що модуль Юнга і щільність матеріалу оболонки є функціями нормальної координати. Чисельно реалізовані частотні рівняння та побудовані характерні криві залежностей.

Ключові слова: *підкріплена оболонка, варіаційний принцип, рідина, вільне коливання, анізотропна оболонка*.

УДК 539.3

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КОЛЕБАНИЯ АНИЗОТРОПНОЙ НЕОДНОРОДНОЙ ПОДКРЕПЛЕННОЙ ПЕРЕКРЕСТНЫМИ РЕБРАМИ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ С ЖИДКОСТЬЮ

Исследованы свободные колебания анизотропной неоднородной подкрепленной перекрестными системами ребер цилиндрической оболочки, контактирующей с движущейся жидкостью. Используя вариационный принцип Гамильтона – Остроградского построена система уравнений её движения. Для учета неоднородности материала оболочки по толщине принимается, что модуль Юнга и плотность материала оболочки являются функциями нормальной координаты. Численно реализованы частотные уравнения и построены характерные кривые.

Ключевые слова: *подкрепленная оболочка, вариационный принцип, жидкость, свободное колебание, анизотропная оболочка*.

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