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VIBRATIONS OF A LONGITUDINALLY STIFFENED, LIQUID-FILLED CYLINDRICAL SHELL IN LIQUID

In the paper we study free vibrations of a longitudinally stiffened, viscous liquid-filled orthotropic cylindrical shell in ideal liquid. The Navier – Stokes linearized equation is used to describe the motion of the internal viscous liquid, the motion of the external liquid is described by a wave equation written in the potential by perturbed velocity. Frequency equation of a longitudinally stiffened orthotropic, viscous liquid-contacting cylindrical shell is obtained on the basis of the Hamilton – Ostrogradsky principle of stationarity of action. Characteristic curves of dependence are constructed.

Keywords: *free vibrations; shell; ideal liquid; viscous liquid; stress; stiffening; variational principle.*

Introduction. In [1], free vibration of an orthotropic, soil-contacting cylindrical shell inhomogeneous in thickness and stiffened with annular ribs, is studied. Using the Hamilton – Ostrogradsky variational principle, a system of equations of motion of a soil-contacting orthotropic cylindrical shell inhomogeneous in thickness and stiffened with annular ribs, is constructed. To account for heterogeneity of the shell material in thickness it is accepted that the Young modulus and the shell material density are the functions of normal coordinate. Using the Hamilton – Ostrogradsky variational principle the frequency equations are structured and implemented numerically. In the calculation process, linear and parabolic laws are accepted for the heterogeneity function. The characteristic curves of dependence are constructed.

The paper [5] was devoted to investigation of one of the dynamical strength characteristics, the frequency of natural vibrations of an inhomogeneous orthotropic, flowing liquid-filled cylindrical shell made of a fiberglass and stiffened with annular ribs under Navier conditions. The results of calculations of natural frequency of vibrations were represented in the form of dependence of the speed of flowing liquid on the amount of stiffening elements for different values of wave formation parameters and different ratios of elasticity module.

The paper [6] represents the results of finding the frequencies of free vibrations of a structurally anisotropic flowing liquid-filled cylindrical shell made of a fiberglass and stiffened with annular ribs under Navier boundary conditions. The results of calculations of natural frequencies of vibrations are given in the form of dependences on the winging angle of the fiberglass for a shell made of a tissue fiberglass and on the speed of flowing liquid for

different values of wave formation parameters and various ratios between the parameters characterizing geometrical sizes of the shell.

The supports formed by the combination of cylindrical panels are used in bridge construction [4]. To save the material, the interior area of the support is filled with soil. Such supports are exposed to different nature forces. One of such forces is a force generated on the surface of cylindrical panels that form supports during flood flow. Under the action of these forces the support is exposed to forced vibration. Therefore, to study the supports formed from combination of cylindrical panels with regard to viscosity and heterogeneity of soil, orthotropic character of panels is of great practical importance. In the paper, based on the Hamilton – Ostrogradsky variational principle, we study forced vibrations of a vertical retaining wall consisting of three orthotropic cylindrical panels contacting with viscous-elastic, heterogeneous soil, obtain analytic expressions to calculate the displacements of the points of cylindrical panels and structure characteristically curves. Account of heterogeneity of soil is performed by accepting its rigidity coefficients as a function of coordinate. It is assumed that the Poisson ratio is constant.

In the paper [9] natural vibrations frequency of the system that consisting of a solid medium-filled elastic-plastic orthotropic cylindrical shell strengthened with discretely distributed rings established on a plane perpendicular to its axis are studied. Utilizing the Hamilton – Ostrogradsky principle, a frequency equation for determining vibration frequencies of the system the following consideration was created; its roots were obtained by mathematical method.

In the paper [3] free vibrations of an orthotropic, laterally stiffened, ideal fluid-filled cylindrical shell inhomogeneous in thickness and in circumferential direction is studied. Using the Hamilton – Ostrogradsky variational principle, the systems of equations of the motion of an orthotropic, ideal fluid filled cylindrical shell stiffened in thickness and circumference, are constructed.

Problem definition. We consider free vibrations of a longitudinally stiffened viscous liquid-filled cylindrical shell in infinite ideal liquid (Fig. 1).

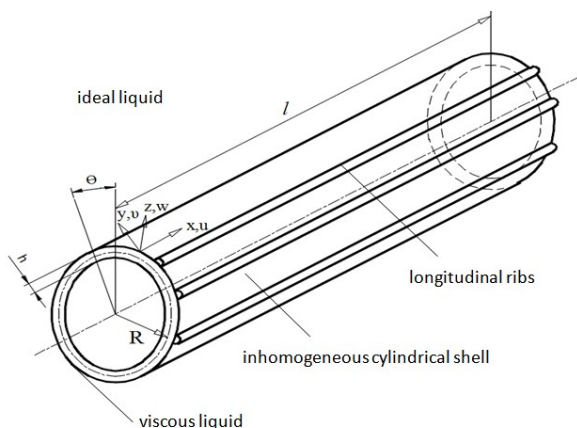


Fig.1 – Longitudinally stiffened inhomogeneous cylindrical shell

The equation of motion of a longitudinally stiffened orthotropic, liquid-filled cylindrical shell in liquid, is obtained on the basis of Hamilton – Ostrogradsky principle of stationarity of action

$$\delta W = 0, \quad (1)$$

where $W = \int_{t'}^{t''} \Pi dt$ is Hamilton's action, t' and t'' are the given arbitrary moments of time. Here

$$\Pi = A_0 + \sum_{i=1}^{k_1} A_n + A_m + A_j. \quad (2)$$

where A_0 is the total energy of the cylindrical shell, A_n is the total energy of the i -th longitudinal bar, k_1 is the moment of longitudinal ribs, A_m and A_j are potential energies of external surface loads acting as viewed from ideal and viscous liquids and applied to the shell and are determined as a work performed by these loads when taking the system from the deformed state to the initial undeformed one and is represented in the form:

$$A_m = -R \int_0^l \int_0^{2\pi} q_{zm} w dx d\theta \quad (3)$$

$$A_j = -R \int_0^L \int_0^{2\pi} (q_x u + q_y v + q_z w) dx d\theta. \quad (4)$$

Here q_x, q_y, q_z are the load intensity acting on the shell as viewed from the viscous liquid, q_{zm} is the load intensity acting on the shell as viewed from ideal liquid.

The expressions for A_0 and A_n are of the form [2]:

$$A_0 = \frac{1}{2} \int_{-h/2}^{h/2} f_1(z) dz \iint \left\{ \tilde{b}_{11} \varepsilon_{11}^2 + 2\tilde{b}_{12} \varepsilon_{11} \varepsilon_{22} + \tilde{b}_{22} \varepsilon_{22}^2 + \tilde{b}_{66} \varepsilon_{12}^2 \right\} f_2(x) dx dy + \int_{-h/2}^{h/2} f_1(z) dz \times \iint \left[\tilde{\rho} \left(\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) \right] f_2(x) dx dy. \quad (5)$$

$$J = \frac{1}{2} \sum_{j=1}^{k_1} \int_{x_1}^{x_2} \left[\tilde{E}_i F_i \left(\frac{\partial u_i}{\partial x} \right)^2 + \tilde{E}_i J_{yi} \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 + \tilde{E}_i J_{zi} \left(\frac{\partial^2 \vartheta_i}{\partial x^2} \right)^2 + \tilde{G}_i J_{kpi} \left(\frac{\partial \phi_{kpi}}{\partial x} \right)^2 \right] dx + \quad (6)$$

$$+ \sum_{i=1}^{k_1} \rho_i F_i \int_{x_1}^{x_2} \left[\left(\frac{\partial u_i}{\partial t} \right)^2 + \left(\frac{\partial v_i}{\partial t} \right)^2 + \left(\frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left(\frac{\partial \phi_{kpi}}{\partial t} \right)^2 \right] dx.$$

In the expressions (5) and (6)

$$\begin{aligned} \varepsilon_{11} &= \frac{\partial u}{\partial x}; \quad \varepsilon_{22} = \frac{\partial \vartheta}{\partial y} + w; \quad \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x}. \\ \tilde{b}_{11} &= \frac{E_1}{1-\nu_1\nu_2}; \quad \tilde{b}_{22} = \frac{E_2}{1-\nu_1\nu_2}; \quad \tilde{b}_{12} = \frac{\nu_1 E_1}{1-\nu_1\nu_2} = \frac{\nu_2 E_2}{1-\nu_1\nu_2}; \quad \tilde{b}_{66} = G \end{aligned}$$

are the basic elasticity module of the homogeneous, orthotropic material of the shell, the displacements of the shell points; $\tilde{\rho}$ is the density of the homogeneous shell material; $f_1(z), f_2(x)$ are the inhomogeneity functions in the direction of normal and generatrix of the shell, respectively [8], ν_1, ν_2 are the Poisson ratios; E_1, E_2 are the Young modulus of the shell material in coordinate directions of the axes x, y , respectively; h is the shell thickness; u_i, ϑ_i, w_i are the displacements of the bars points used in stiffening; F_i are the areas of cross sections of the i -th bar attached to the shell in the direction of generatrix; \tilde{E}_i is an elasticity modulus when the i -th bar attached to the cylindrical shell is stretched in the direction of the generatrix; J_{yi}, J_{zi} are the inertia moments of the i -th bar with respect to the axis passing through the center of gravity of the cross-section; J_{kpi} are inertia moments when the i -th bar is twisted; t is time, ρ_i is the density of the material of the i -th bar; $\varphi_i(x), \varphi_{kpi}(x)$ are the torsional angle of the bar cross-section and through the shell displacement are expressed as follows

$$\varphi_{kpi}(x) = \varphi_2(x, y_i) = - \left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R} \right) \Big|_{y=y_i}.$$

Assuming that the basic flow rate equal U and deviations from this rate are small, we use the wave equation for disturbed velocity potential $\tilde{\Phi}$ with respect to [7, 10]

$$\Delta \tilde{\Phi} - \frac{1}{a_0^2} \left(\frac{\partial^2 \tilde{\Phi}}{\partial t^2} + 2U \frac{\partial^2 \tilde{\Phi}}{R \partial x \partial t} + U^2 \frac{\partial^2 \tilde{\Phi}}{\partial x^2} \right) = 0. \quad (7)$$

To describe the motion of external viscous liquid we use the Navier – Stokes linearized equation for viscous compressible liquid [10]:

$$\rho_m \frac{\partial \bar{\vartheta}}{\partial t^2} = -grad p + \frac{1}{3} grad div \bar{\vartheta} + \bar{\mu} \nabla^2 \bar{\vartheta}, \quad (8)$$

where $\vec{\mathfrak{g}}(\mathfrak{g}_x, \mathfrak{g}_r, \mathfrak{g}_\theta)$ is a vector of velocity of an arbitrary point of liquid, p is pressure at arbitrary point of liquid, ρ_m is liquid density.

The expression of the total energy of the system (2), the equation of motion of ideal liquid (7) and viscous liquid (8) are supplemented by contact conditions. On the contact surface of a shell-ideal liquid we observe continuity of radial velocities and pressures. The condition of impermeability or smoothness of flow at the shell wall is of the form [10]

$$\mathfrak{g}_r \Big|_{r=R} = \frac{\partial \tilde{\Phi}}{\partial r} \Big|_{r=R} = - \left(\omega \frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} \right) \Big|_{r=R}. \quad (9)$$

Equality of radial pressures as viewed from liquid on the shell:

$$q_{zm} = -\tilde{p} \Big|_{r=R}. \quad (10)$$

On the contact surface of a shell-viscous liquid continuity of radial velocities and pressures is observed, i.e. for $r = R$ there will be *

$$\mathfrak{g}_x = \frac{\partial u}{\partial t}, \mathfrak{g}_\theta = \frac{\partial \tilde{\mathfrak{g}}}{\partial t}, \mathfrak{g}_r = \frac{\partial w}{\partial t} \quad (11)$$

$$q_x = -\sigma_{rx}, q_y = -\sigma_{r\theta}, q_z = -p \quad (12)$$

where $\sigma_{rx}, \sigma_{r\theta}$ are viscous forces [1].

It is considered that the rigid contact conditions between the shell and bars are satisfied:

$$u_i(x) = u(x, y_i) + h_i \varphi_1(x, y_i), \quad \mathfrak{g}_i(x) = \mathfrak{g}(x, y_i) + h_i \varphi_2(x, y_i), \\ w_i(x) = w(x, y_i), \quad \varphi_i(x) = \varphi_1(x, y_i), \quad \varphi_{kpi}(x) = \varphi_2(x, y_i); \quad h_i = 0, 5h + H_i^1,$$

where H_i^1 is the distance from the i -th bar to the surface of the cylindrical shell; h_i is the thickness of the i -th longitudinal bar;

$$\phi_i(x) = \phi_1(x, y_i) = - \frac{\partial w}{\partial x} \Big|_{y=y_i}.$$

Problem solution. We represent the solution of the Navier – Stokes equation through the scalar potential ϕ and vector potential $\vec{\Psi}$ in the form

$$\mathfrak{g} = \text{grad} \phi + \text{rot} \vec{\Psi}. \quad (13)$$

Substituting (7) in (6), we get:

$$\rho_m \frac{\partial (\text{grad} \phi + \text{rot} \vec{\Psi})}{\partial t} = -\text{grad} p + \frac{1}{3} \mu \text{grad} \text{div} \vec{\mathfrak{g}} + \bar{\mu} \Delta \vec{\mathfrak{g}}. \quad (14)$$

From (13) we easily get

$$\operatorname{div} \vec{\mathfrak{G}} = \Delta \phi; \operatorname{grad} \operatorname{div} \vec{\mathfrak{G}} = \operatorname{grad} \Delta \phi.$$

Using the vector identity $\operatorname{rot} \operatorname{rot} \vec{\mathfrak{G}} = \operatorname{grad} \operatorname{div} \vec{\mathfrak{G}} - \Delta \vec{\mathfrak{G}}$, we can write

$$\Delta \vec{\mathfrak{G}} = \operatorname{grad} \operatorname{div} \vec{\mathfrak{G}} - \operatorname{rot} \operatorname{rot} \vec{\mathfrak{G}} = \operatorname{grad} \Delta \phi - \operatorname{rot} \operatorname{rot} \vec{\mathfrak{G}}.$$

Using (7) we find

$$\operatorname{rot} \operatorname{rot} \vec{\mathfrak{G}} = \operatorname{rot} \operatorname{rot} (\operatorname{grad} \phi + \operatorname{rot} \vec{\psi}) = \operatorname{rot} \operatorname{rot} \operatorname{grad} \phi + \operatorname{rot} (\operatorname{rot} \operatorname{rot} \vec{\psi}) = -\operatorname{rot} \Delta \vec{\psi};$$

$$\operatorname{grad} \operatorname{div} \vec{\mathfrak{G}} = \operatorname{grad} (\Delta \phi).$$

Substituting these relations in the equation of motion (8) we find

$$\rho_m \frac{\partial}{\partial t} (\operatorname{grad} \phi) + \operatorname{grad} p - \frac{4}{3} \bar{\mu} \operatorname{grad} p - \frac{4}{3} \bar{\mu} \operatorname{grad} \Delta \phi - \bar{\mu} \operatorname{rot} \Delta \vec{\psi} + \rho_m \frac{\partial}{\partial t} \operatorname{rot} \vec{\psi} = 0$$

or

$$\operatorname{grad} \left(\rho_m \frac{\partial \phi}{\partial t} + p - \frac{4}{3} \bar{\mu} \Delta \phi \right) + \operatorname{rot} \left(-\bar{\mu} \Delta \vec{\psi} + \rho_m \frac{\partial \vec{\psi}}{\partial t} \right) = 0.$$

This equation will be satisfied if we assume

$$\rho_m \frac{\partial \phi}{\partial t} + p - \frac{4}{3} \bar{\mu} \Delta \phi = 0; \quad (15)$$

$$-\bar{\mu} \Delta \vec{\psi} + \rho_m \frac{\partial \vec{\psi}}{\partial t} = 0. \quad (16)$$

Thus, the particular solution of equation (8) can be obtained based on the particular solutions (15) and (16). From (9) and (10) it can be seen that for finding the potentials ϕ and $\vec{\psi}$ we need to know the pressure p . We illustrate it on an example, when the liquid is viscous Newtonian. In this case to the system of Navier – Stokes linearized equations (8) that contains five unknowns, three components of velocity $\mathfrak{G}_x, \mathfrak{G}_r, \mathfrak{G}_\theta$, pressure p and density

ρ_m , we add the continuity equation $\frac{\partial \rho}{\partial t} + \rho_m \operatorname{div} \vec{\mathfrak{G}} = 0$ and the formula closing

the system in the form $\frac{\partial p}{\partial \rho} = a_*^2$. In the monograph [10] after some

transformations the following linearized wave equation is obtained

$$\frac{1}{a_*^2} \frac{\partial^2 p}{\partial t^2} = \nabla^2 \left(p + \frac{4\bar{\mu}}{3\rho_m a_*^2} \frac{\partial p}{\partial t} \right). \quad (17)$$

The solution of equation (11) is of the form

$$p = (p_0 J_n(\lambda r) + c_0 Y_n(\lambda r)) \exp i(kx + n\theta + \omega t) \quad (18)$$

where

$$\lambda = \sqrt{\frac{\omega^2}{a_*^2 \left(1 + i \frac{4\bar{\mu}\omega}{3\rho_m a_*^2} \right)}} - k^2, \quad I_n, Y_n$$

are first and second kind Bessel functions of order n , where n is the amount of waves along the circumference; k is a wave number or a constant propagated phase, where $k = \tilde{m}\pi/L$, \tilde{m} is the amount of longitudinal waves in the shell, the value ω characterizes cyclic frequency of the wave; $\bar{\mu}$ is a dynamical viscosity factor; ρ_m is liquid's density in unperturbed state; a_* is the velocity of propagated small perturbations in liquid; p_0, c_0 are constants.

Assuming the function p bounded as $r \rightarrow \infty$, we find $p_0 = 0$ and then finally

$$p = c_0 Y_n(\lambda r) \exp i(kx + n\theta + \omega t). \quad (19)$$

From (15), for finding ϕ we get the equation

$$\Delta\phi - \frac{3\rho_m}{4\bar{\mu}} \frac{\partial\phi}{\partial t} = c_0 Y_n(\lambda r) \exp i(kx + n\theta + \omega t). \quad (20)$$

The solution of the homogeneous equation (20) is of the form

$$\phi = C_1 I_n(\tilde{k}r) + C_2 K_n(\tilde{k}r),$$

where $\tilde{k} = \sqrt{k^2 + 3i\omega\rho_m/4\bar{\mu}}$, $I_n(\tilde{k}r)$, $K_n(\tilde{k}r)$ are the first and second kind modified Bessel functions of order n , respectively; C_1, C_2 are constants. By means of the method of variation of constants, we can write the solution of the equation in the form

$$\phi(r) = p_0 f(r) + \mu_1 K_n(\tilde{k}r), \quad (21)$$

where

$$\Delta(r) = I_n(\tilde{k}r) K'_n(\tilde{k}r) - I'_n(\tilde{k}r) K_n(\tilde{k}r);$$

$$f(r) = I_n(\tilde{k}r) \int_r^R \Delta^{-1}(\xi) J_n(\lambda\xi) K_n(\tilde{k}\xi) d\xi + K_n(\tilde{k}r) \int_0^r \Delta^{-1}(\xi) J_n(\lambda\xi) I_n(\tilde{k}\xi) d\xi.$$

The equation with respect to the componenets of the vector $\bar{\Psi}(\psi_1, \psi_2, \psi_3)$ is of the form $\bar{\Delta}\bar{\Psi} = \frac{\rho_m}{\bar{\mu}} \frac{\partial \bar{\Psi}}{\partial t}$

or

$$\psi_i''(r) + \frac{1}{r} \psi_i'(r) - \left(k^2 + \frac{i\omega \rho_m}{\bar{\mu}} + \frac{n^2}{r^2} \right) \psi_i(r) = 0. \quad (22)$$

The solution of equation (22) corresponding to the problem under consideration, is of the form:

$$\psi_i = \mu_2 Y_n(qr) \quad (i = 1, 2, 3) \quad (23)$$

here $q = \sqrt{k^2 + i\omega/\bar{\mu}}$.

Using (13), (17) and (19), for the components of velocity vector we get:

$$\begin{aligned} v_x &= \left[-\frac{k\omega}{\rho_m a_*^2} p_0 f(r) + ikJ_n(kr)\mu_1 + (inJ_n(qr) - qJ_n'(qr))\mu_2 \right] \exp i(kx + n\theta + \omega t); \\ v_\theta &= \left[-\frac{n\omega}{\rho_m a_*^2} p_0 f(r) + inJ_n(kr)\mu_1 + i\left(k - \frac{n}{R}\right)J_n(qr)\mu_2 \right] \exp i(kx + n\theta + \omega t); \\ v_r &= \left[\frac{i\omega}{\rho_m a_*^2} p_0 f'(r) + kJ_n'(kr)\mu_1 + (qJ_n'(qr) - ikJ_n(qr))\mu_2 \right] \exp i(kx + n\theta + \omega t). \end{aligned} \quad (24)$$

By means of the viscosity force formula [1] we find:

$$\begin{aligned} \sigma_{rx} &= \bar{\mu} \left[-\frac{2k\omega}{\rho_m a_*^2} f'(r) p_0 + 2ikJ_n'(kr)\mu_1 + \right. \\ &\quad \left. + \left(-k\left(k - \frac{n}{R}\right)J_n(qr) + \frac{in}{R}J_n'(qr) - J_n''(qr) \right) \mu_2 \right] \exp i(kx + n\theta + \omega t); \\ \sigma_{r\theta} &= \bar{\mu} \left[-\frac{2n\omega}{R\rho_m a_*^2} f'(r) p_0 + \frac{2in}{R}J_n'(kr)\mu_1 + \right. \\ &\quad \left. + \left(i\left(k - \frac{n}{R}\right)J_n(qr) - ikJ_n'(qr) + J_n''(qr) \right) \mu_2 \right] \exp i(kx + n\theta + \omega t); \\ \sigma_{rr} &= p_0 J_n(\lambda r) \exp i(kx + n\theta + \omega t). \end{aligned} \quad (25)$$

Using contact conditions (12) and expressions (25), we find the forces q_x, q_y, q_z acting on the shell as viewed from viscous liquid

$$\begin{aligned}
 q_x = \bar{\mu} & \left[-\frac{2k\omega}{\rho_m a_*^2} f'(r) p_0 + 2ikJ'_n(kr)\mu_1 + \right. \\
 & \left. + \left(-k \left(k - \frac{n}{R} \right) J_n(qr) + \frac{in}{R} J'_n(qr) - J''_n(qr) \right) \mu_2 \right] \exp i(kx + n\theta + \omega t); \\
 q_z = p_0 J_n(\lambda R) \exp i(kx + n\theta + \omega t); \\
 q_y = \bar{\mu} & \left[-\frac{2n\omega}{R\rho_m a_*^2} f'(R) p_0 + \frac{2in}{R} J'_n(kr)\mu_1 + \right. \\
 & \left. + \left(i \left(k - \frac{n}{R} \right) J_n(qr) - ikJ'_n(qr) + J''_n(qr) \right) \mu_2 \right] \exp i(kx + n\theta + \omega t). \tag{26}
 \end{aligned}$$

We will look for the displacements of the shell points in the form

$$\begin{aligned}
 u = u_{0kn} \exp i(kx + n\theta + \omega t); \quad \tilde{\vartheta} = \vartheta_{0kn} \exp i(kx + n\theta + \omega t) \tag{27} \\
 w = w_{0kn} \exp i(kx + n\theta + \omega t).
 \end{aligned}$$

Here $u_{0kn}, \vartheta_{0kn}, w_{0kn}$ are unknown constants.

Contact conditions (11) imply a system of algebraic equations with respect to unknown constants $p_0, \mu_1, \mu_2, u_{0kn}, \vartheta_{0kn}, w_{0kn}$. This system allows to express the constants and μ_2 by the constants $u_{0kn}, \vartheta_{0kn}, w_{0kn}$. Then we get:

$$\begin{aligned}
 q_x = \bar{\mu} i \omega \Delta^{-1} & \left[\left(-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{11} + 2ikJ'_n(kR) \Delta_{12} + \left(-k \left(k - \frac{n}{R} \right) \times \right. \right. \right. \\
 & \times J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \Big) \Delta_{13} \Big) u_{0kn} + \\
 & + \left(-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{21} + 2ikJ'_n(kR) \Delta_{22} + \vartheta_{0kn} + \right. \\
 & + \left(-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{31} + 2ikJ'_n(kR) \Delta_{32} + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \right. \right. \\
 & \left. \left. + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{33} \right) w_{0kn} \Big] \exp i(kx + n\theta + \omega t); \tag{28}
 \end{aligned}$$

$$\begin{aligned}
q_y = & \bar{\mu} i \omega \Delta^{-1} \left[\left(-\frac{2k\omega}{R\rho_m a_*^2} f'(R) \Delta_{11} + \frac{2in}{R} J'_n(kR) \Delta_{12} + \left(i \left(k - \frac{n}{R} \right) \times \right. \right. \right. \\
& \times J_n(qR) - ikJ'_n(qR) + J''_n(qR) \Big) \Delta_{13} \Big) u_{0kn} + \left(-\frac{2k\omega}{R\rho_m a_*^2} f'(R) \Delta_{21} + \right. \\
& + \frac{2in}{R} J'_n(kR) \Delta_{22} + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + J''_n(qR) \right) \Delta_{23} \Big) \vartheta_{0kn} + \\
& + \left(-\frac{2n\omega}{R\rho_m a_*^2} f'(R) \Delta_{31} + \frac{2in}{R} J'_n(kR) \Delta_{32} + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - \right. \right. \\
& \left. \left. - ikJ'_n(qR) + J''_n(qR) \right) \Delta_{33} \right) w_{0kn} \Big] \exp i(kx + n\theta + \omega t);
\end{aligned}$$

$$q_z = J_n(\lambda R) i \omega \Delta^{-1} (\Delta_{11} u_{0kn} + \Delta_{21} \vartheta_{0kn} + \Delta_{31} w_{0kn}) \exp i(kx + n\theta + \omega t),$$

where Δ is the main determinant, $\Delta_{sp}(s, p = 1, 2, 3)$ are auxiliary determinants of this system. These determinants are given in [6].

We can calculate the work performed by these loads when taking the system from the deformed state to the initial undeformed state.

$$\begin{aligned}
A_j = & \frac{2\bar{\mu} i \omega \Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{\frac{\pi ni}{2}} - 1 \right) e^{i\omega t} \left\{ \left[-\frac{2k\omega}{\rho_0 a_*^2} f'(R) \Delta_{11} + 2ikJ'_n(kR) \Delta_{12} + \right. \right. \\
& + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{13} \Big] u_{0kn}^2 + \\
& + \left[-\frac{2n\omega}{R\rho_0 a_*^2} f'(R) \Delta_{21} + \frac{2in}{R} J'_n(kR) \Delta_{22} + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + \right. \right. \\
& + J''_n(qR) \Big) \Delta_{23} \Big] \vartheta_{0kn}^2 + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{31} w_{0kn}^2 + \left[-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{21} + \right. \\
& + 2ikJ'_n(kR) \Delta_{22} + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{23} - \\
& - \frac{2n\omega}{R\rho_m a_*^2} f'(R) \Delta_{11} + \frac{2in}{R} J'_n(kR) \Delta_{12} + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) \right. \\
& + J''_n(qR) \Big) \Delta_{13} \Big] u_{0kn} \vartheta_{0kn} + \left[-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{31} + 2ikJ'_n(kR) \Delta_{32} + \right.
\end{aligned} \tag{29}$$

$$\begin{aligned}
& + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{33} + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{31} \Big] u_{0kn} w_{0kn} + \\
& + \left[-\frac{2n\omega}{R\rho_m a_*^2} f'(R) \Delta_{33} + \frac{2in}{R} J'_n(qR) \Delta_{32} + \right. \\
& \left. + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ik J'_n(qR) + J''_n(qR) \right) \Delta_{33} + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{21} \right] \Big] g_{0kn} w_{0kn}.
\end{aligned}$$

We look for the perturbed velocities potential ϕ in the form:

$$\tilde{\phi}(x, r, \theta, t) = f(r) \exp i(kx + n\theta + \omega t). \quad (30)$$

Using (30), from equation (7) we have [10]:

$$\tilde{\phi} = -\varphi_{\alpha n} \left(\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} \right), \quad (31)$$

$$\tilde{p} = \varphi_{\alpha n} \rho^* \left(\frac{\partial^2 w}{\partial t^2} + 2U \frac{\partial^2 w}{R \partial x \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} \right), \quad (32)$$

where

$$\varphi_{\alpha n} = \begin{cases} \frac{K_n(\beta r)}{K'_n(\beta R)}, & M_1 < 1 \\ \frac{N_n(\beta r)}{N'_n(\beta R)}, & M_1 > 1 \\ \frac{r^n}{nR^{n-1}}, & M_1 = 1. \end{cases}$$

Here

$$M_1 = \frac{U + \omega R / k}{a_0}; \quad \beta^2 = R^{-2} (1 - M_1^2) k^2; \quad \beta_1^2 = R^{-2} (M_1^2 - 1) k^2 K_n$$

is a second kind n -th order modified Bessel function; N_n are n -th order Neumann functions; ρ_* is density of liquid.

Proceeding from condition (10), using (32) we have

$$q_{zm} = \tilde{\phi}_{\alpha n} \rho^* \left(\frac{\partial^2 w}{\partial t^2} + 2U \frac{\partial^2 w}{R \partial x \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} \right), \quad (33)$$

where $\tilde{\phi}_{\alpha n} = \varphi_{\alpha n} \Big|_{r=R}$.

We can calculate the work performed by q_z when taking the system from the deformed state to the initial under formed state

$$A_m = \frac{2R\tilde{\phi}_{an}\rho^*}{nk} \left(e^{ikl} - 1 \right) \left(1 - e^{in\pi} \right) \left(\omega^2 + 2Uk\omega + k^2U^2 \right).$$

Substitute approximation (27) in functional Π (2) and considering that in this functional $x_1 = 0$, $x_2 = l$, $y_1 = 0$, $y_2 = 2\pi$, $t' = 0$, $t'' = \pi/\omega$, we integrate with respect to x, y and t . Then instead of the functional Π we get the function W of the required values $u_{0kn}, \vartheta_{0kn}, w_{0kn}$. The stationary value of the obtained function is determined by the following system of equations:

$$\frac{\partial W}{\partial u_{0kn}} = 0; \quad \frac{\partial W}{\partial \vartheta_{0kn}} = 0; \quad \frac{\partial W}{\partial w_{0kn}} = 0 \quad (34)$$

Or

$$\begin{aligned} & \left\{ -\frac{4i}{n} \left(k^2 \tilde{b}_{11} + \frac{n^2 \tilde{b}_{66}}{R^2} \right) ST \left(1 - e^{in\pi} \right) - 2\omega^2 \left(\tilde{\rho} \cdot S \cdot T + L \sum_{i=1}^{k_1} \rho_i F_i \sin^2 n\theta_i \right) + \right. \\ & + \frac{4\tilde{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \left[-\frac{2k\omega}{\rho_0 a_*^2} f'(R) \Delta_{11} + 2ikJ'_n(kR) \Delta_{12} + \right. \\ & + \left. \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{13} \right] + \\ & + \frac{l}{2} \sum_{i=1}^{k_1} E_i F_i k^2 \sin^2 n\theta_i \left. \right\} u_{0kn} + \left\{ -\frac{4i}{n} \left(\frac{2nk\tilde{b}_{12}}{R} + \frac{nk^2}{R} \tilde{b}_{66} \right) ST (1 - e^{in\pi}) + \right. \\ & + \frac{4\tilde{\mu}i\omega\Delta^{-1}}{nk} (e^{ikl} - 1) (e^{in\pi} - 1) \left[-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{21} + 2ikJ'_n(kR) \Delta_{22} + \right. \\ & + \left. \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{23} - \frac{2n\omega}{R\rho_m a_*^2} f'(R) \Delta_{11} + \right. \\ & + \frac{2in}{R} J'_n(kR) \Delta_{12} + \left. \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + J''_n(qR) \right) \Delta_{13} \right] \left. \right\} \vartheta_{0kn} + \\ & + \left\{ -\frac{4ik\tilde{b}_{12}}{nR} ST (1 - e^{in\pi}) + \frac{4\tilde{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \times \right. \\ & \times \left[\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{31} + 2ikJ'_n(kR) \Delta_{32} + \left(k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - \right. \right. \\ & \left. \left. - J''_n(qR) \right) \Delta_{33} \right] \left. \right\} w_{0kn} = 0; \quad (35) \end{aligned}$$

$$\begin{aligned}
& \times \left[-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{21} + 2ikJ'_n(kR) \Delta_{22} + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \right. \right. \\
& \quad \left. \left. + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{23} - \frac{2n\omega}{R \rho_m a_*^2} f'(R) \Delta_{11} + \frac{2in}{R} J'_n(kR) \Delta_{12} + \right. \\
& \quad \left. + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + J''_n(qR) \right) \Delta_{13} \right] u_{0kn} + \\
& \quad + \left\{ -\frac{4i}{n} \left(\frac{n^2 \tilde{b}_{22}}{R^2} + k^2 \tilde{b}_{66} \right) ST \left(1 - e^{in\pi} \right) + \frac{4\bar{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \times \right. \\
& \quad \times \left[-\frac{2n\omega}{R \rho_m a_*^2} f'(R) \Delta_{21} + \frac{2in}{R} J'_n(kR) \Delta_{22} + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + \right. \right. \\
& \quad \left. \left. + J''_n(qR) \right) \Delta_{23} + \frac{l}{2} \sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} k^2 \cos^2 n\theta_i - \omega^2 \left(\tilde{\rho} \cdot S \cdot T + L \sum_{i=1}^{k_1} \rho_i F_i \sin^2 n\theta_i \right) + \right. \\
& \quad \left. + L \sum_{i=1}^{k_1} \rho_i F_i \cos^2 n\theta_i + i \sum_{i=1}^{k_1} \frac{\rho_i J_{kpi} \cos^2 n\theta_i}{R^2} \right] \Bigg\} \vartheta_{0kn} + \left\{ \frac{\sin \tilde{b}_{22}}{R^2} ST \left(1 - e^{in\pi} \right) \times \right. \\
& \quad \times \left[-\frac{2n\omega}{R \rho_0 a_*^2} f'(R) \Delta_{31} + \frac{2in}{R} J'_n(kR) \Delta_{32} \frac{4\bar{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \times \right. \\
& \quad \times \left[-\omega^2 L \sum_{i=1}^{k_1} \frac{m \rho_i J_{kpi} \cos^2 n\theta_i}{R^2} + \frac{L}{2} \sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} k^2 m \cos^2 n\theta_i + \right. \\
& \quad \left. \left. + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ikJ'_n(qR) + J''_n(qR) \right) \Delta_{33} + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{21} \right] \right\} w_{0kn} = 0; \\
& \quad \left\{ -\frac{4k\tilde{b}_{12}}{nR} ST \left(1 - e^{in\pi} \right) + \frac{4\bar{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \right\} \left[-\frac{2k\omega}{\rho_m a_*^2} f'(R) \Delta_{31} + \right. \\
& \quad \left. + 2ikJ'_n(kR) \Delta_{32} + \left(-k \left(k - \frac{n}{R} \right) J_n(qR) + \frac{in}{R} J'_n(qR) - J''_n(qR) \right) \Delta_{33} + \right. \\
& \quad \left. + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{31} \right] u_{0kn} + \left\{ -\frac{4\tilde{b}_{22}}{R^2} ST \left(1 - e^{in\pi} \right) + \frac{4\bar{\mu}i\omega\Delta^{-1}}{nk} \times \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \left[-\frac{2n\omega}{R\rho_m a_*^2} f'(R) \Delta_{31} + \frac{2in}{R} J'_n(kR) \Delta_{32} - \right. \\
& - \omega^2 l \sum_{i=1}^{k_1} \rho_i \frac{J_{kpi} \cos^2 n\theta_i}{R^2} \cdot n - \frac{L}{2} \sum_{i=1}^{k_1} \frac{G_i J_{kpi}}{R^2} n k^2 \cos^2 n\theta_i + \\
& + \left(i \left(k - \frac{n}{R} \right) J_n(qR) - ik J'_n(qR) + J''_n(qR) \right) \Delta_{33} + J_n(\lambda R) \bar{\mu}^{-1} \Delta_{21} \left. \right] \mathfrak{g}_0 + \\
& + \left\{ \frac{-4i\tilde{b}_{22}}{nR^2} ST(1 - e^{in\pi}) + \frac{4\bar{\mu}i\omega\Delta^{-1}}{nk} \left(e^{ikl} - 1 \right) \left(e^{in\pi} - 1 \right) \left[-2J_n(\lambda R) \bar{\mu}^{-1} \Delta_{31} - \right. \right. \\
& \left. \left. - \omega^2 \left(\tilde{\rho} \cdot S \cdot T + L \sum_{i=1}^{k_1} \rho_i F_i \sin^2 n\theta_i + L \sum_{i=1}^{k_1} \rho_i \frac{J_{kpi} \cos^2 n\theta_i n^2}{R^2} \right) \right] w_{0kn} = 0, \right.
\end{aligned}$$

where $S = \frac{1}{2} \int_{-h/2}^{h/2} f_1(z) dz$, $T = \int_0^l e^{ikx} f_2(x) dx$.

Since the system (35) is a homogeneous algebraic system of linear equations, the necessary and sufficient condition for the existence of its nonzero solution is the equality of its main determinant to zero. As a result we get the following frequency equation

$$\begin{vmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{vmatrix} = 0, \quad (36)$$

where φ_{ij} ($i, j = 1, 2, 3$) are the coefficients for the unknowns $u_{0kn}, \mathfrak{g}_{0kn}, w_{0kn}$ in the system.

Conclusions. Equation (36) was calculated by the numerical method. The parameters contained in the solution of the problem are:

$$\begin{aligned}
& \tilde{b}_{11} = 18,3 \text{ HPa}; \quad \tilde{b}_{12} = 2,77 \text{ HPa}; \quad \tilde{b}_{22} = 25,2 \text{ HPa}; \quad \tilde{b}_{66} = 3,5 \text{ HPa}; \\
& \rho = \rho_i = 1,85 \cdot 10^4 \frac{\text{H}}{\text{m}^3}; \quad \tilde{E}_i = 6,67 \cdot 10^9 \frac{\text{H}}{\text{m}^2}; \quad v_1 = v_2 = 0,35; \quad a_0 = 1450 \frac{\text{m}}{\text{sec}}; \\
& \rho_m = 1 \cdot 10^4 \frac{\text{H}}{\text{m}^3}; \quad \alpha = 0,4; \quad h = 0,45 \cdot 10^{-3} \text{ m}; \quad \frac{l}{R} = 3; \quad \tilde{m} = 1; \quad R = 1,6 \text{ m}; \\
& n = 8; \quad F_i = 5,2 \cdot 10^{-6} \text{ m}^2; \quad I_{kp,i} = 0,23 \cdot 10^{-12} \text{ m}^4; \quad I_{yi} = 5,1 \cdot 10^{-12} \text{ m}^4; \\
& I_{zi} = 1,3 \text{ mm}^4 \quad f_1(z) = 1 + \alpha \frac{z}{l}; \quad f_2(x) = 1 + \beta \frac{x}{l}.
\end{aligned}$$

The results of calculations were given in Fig. 2 in the form of the dependence of frequency parameter U/a_0 . On the amount of stiffening longitudinal bars k_1 on the shell surface, in Fig. 3. in the form of dependence of the frequency parameter on inhomogeneity parameter in the direction of the generatrix β of the shell, in Fig. 4 in the form of dependence of the frequency parameter on the liquid flow rate U . In the figures, the dotted lines correspond to vibrations of a longitudinally stiffened viscous liquid-filled cylindrical shell in infinite ideal liquid, the solid lines correspond to vibrations of a longitudinally stiffened cylindrical shell in an infinite ideal liquid.

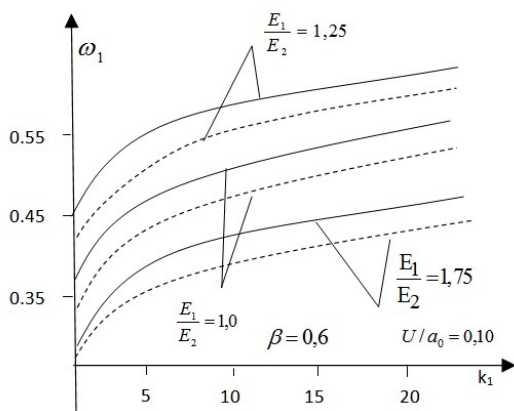


Fig. 2 – Dependence of the frequency parameter on the amount of longitudinal ribs k_1 .

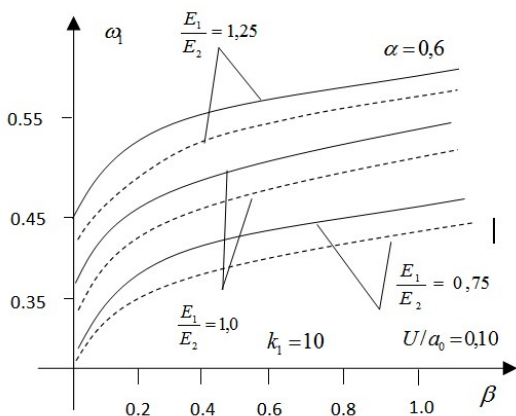


Fig. 3 – Dependence of the frequency parameter on the inhomogeneity parameter β in the longitudinal direction

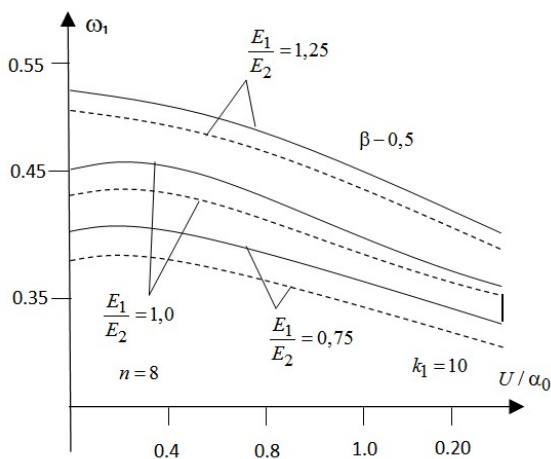


Fig. 4 – Dependence of the frequency parameter on liquid flow rate

From the figures it can be seen that availability of viscous liquid leads to decrease in the value of natural frequency vibrations of the system compared to the frequency of vibrations of the system with any liquid. As can be seen from Fig. 2 with increasing the amount of longitudinal ribs, the value of the frequency parameter increases. As the inhomogeneity parameter increases in the direction of the generatrix of the shell β , as can be seen from Fig. 3 the value of the frequency parameter increases. Furthermore, the value of the frequency parameter increases with increasing orthotropic properties of the cylindrical shell, and decreases with increasing the liquid flow rate (Fig. 4).

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УДК 539.3

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КОЛИВАННЯ В РІДИНІ ПОЗДОВЖНЬО ПІДКРІПЛЕНОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ, ЩО ЗАПОВНЕНА В'ЯЗКОЮ РІДИНОЮ

Досліджено вільні коливання в ідеальній рідині поздовжньо зміцненої ортотропної циліндричної оболонки, що заповнена в'язкою рідиною. Для опису руху внутрішньої в'язкої рідини використовується лінеаризоване рівняння Нав'є – Стокса, рух зовнішньої рідини описується хвильовим рівнянням, записаним в потенціалі обуреної швидкості. На основі принципу стаціонарності дії Гамільтона – Остроградського отримано частотне рівняння поздовжньо підкріпленої ортотропної циліндричної оболонки з зв'язкою рідиною, що контактує з ідеальною рідиною. Побудовано характерні криві залежностей.

Ключові слова: вільні коливання; оболонка; ідеальна рідина; в'язка рідина; напруження; жорсткість; варіаційний принцип.

Розглядаємо вільні коливання в ідеальній рідині поздовжньо зміцненої ортотропної циліндричної оболонки, заповненої в'язкою рідиною. Рівняння руху в рідині ортотропної оболонки, заповненої рідиною, отримано на основі принципу стаціонарності дії Гамільтона – Остроградського

$$\delta W = 0,$$

де $W = \int_{t'}^{t''} \Pi dt$ – дія за Гамільтоном; t' і t'' – задані довільні моменти часу; $\Pi = A_0 + \sum_{i=1}^{k_1} A_n + A_m + A_j$; A_0 – загальна енергія циліндричної оболонки; A_n – загальна енергія i -го поздовжнього ребра; k_1 – момент поздовжніх ребер; A_m і A_j – потенційні енергії зовнішніх поверхневих навантажень від ідеальних і в'язких рідин, які прикладаються до оболонки і визначаються як робота, виконана цими навантаженнями при переведенні системи з деформованого стану в початковий недеформований, і представляється у вигляді

$$A_m = -R \int_0^L \int_0^{2\pi} q_{zm} w dx d\theta, \quad A_j = -R \int_0^L \int_0^{2\pi} (q_x u + q_y v + q_z w) dx d\theta.$$

Тут q_x, q_y, q_z – інтенсивність навантаження, що діє на оболонку з боку в'язкої рідини, q_{zm} – інтенсивність навантаження, яке діє на оболонку з боку ідеальної рідини.

Для опису руху внутрішньої в'язкої рідини використовується лінеаризоване рівняння Нав'є – Стокса, рух зовнішньої рідини описується хвильовим рівнянням, записаним у формі потенціалу збурених швидкостей.

Після спеціальних перетворень одержана однорідна система лінійних алгебраїчних рівнянь. Необхідною та достатньою умовою існування її ненульового розв'язку є рівність її основного детермінанта нулю. В результаті отримуємо таке рівняння частоти

$$\begin{vmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{vmatrix} = 0,$$

де $\varphi_{ij} (i, j = 1, 2, 3)$ – функції невідомих; u_i, ϑ_i, w_i – переміщення стержнів, що використовуються для підкріплення оболонки.

Отримані залежності частоти коливань від кількості поздовжніх підкріплень, параметру неоднорідності та обсягу заповненої рідини.

Наведені числові результати. Проведено порівняльний аналіз параметру частоти для випадку заповнення оболонки ідеальною рідиною. На рис. 1– рис. 3 пунктирні лінії відповідають коливанням поздовжньо зміцненої заповненої в'язкою рідиною циліндричної оболонки, що знаходиться в нескінченній ідеальній рідині, суцільні лінії – коливанням поздовжньо зміцненої циліндричної оболонки в нескінченній ідеальній рідині.

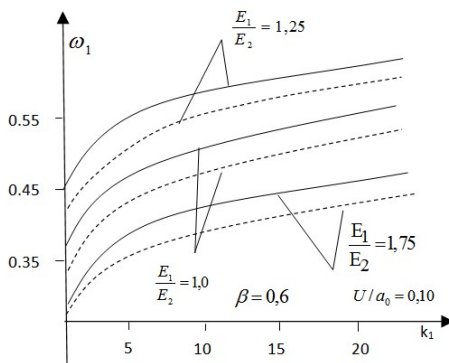


Рис. 1 – Залежність частотного параметра від кількості поздовжніх ребер k_1

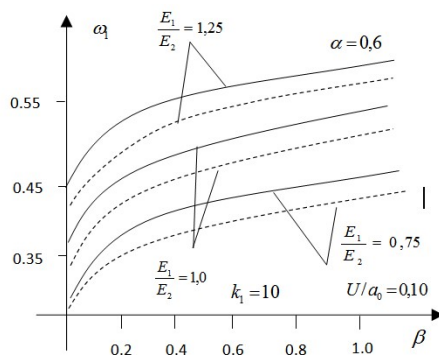


Рис. 2 – Залежність частотного параметра від параметра неоднорідності β в поздовжньому напрямку

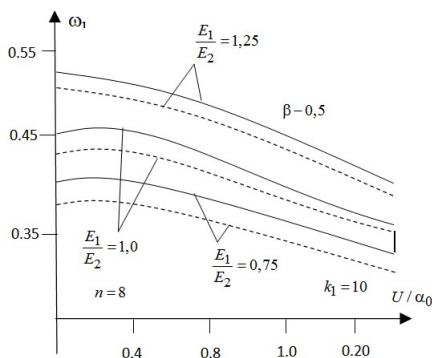


Рис. 3 – Залежність частотного параметра від витрати рідини

З рисунків видно, що наявність в'язкої рідини призводить до зниження значення власної частоти коливань системи у порівнянні з частотою коливань системи з будь-якою іншою рідиною.

Як видно з рис. 1, зі збільшенням кількості поздовжніх ребер значення частотного параметра збільшується.

При збільшенні параметра неоднорідності в напрямку утворювальної лінії оболонки (рис. 2) значення частотного параметра збільшується.

Крім того, значення частотного параметра збільшується з посиленням ортотропних властивостей циліндричної оболонки і зменшується зі збільшенням витрат рідини (рис. 3).

Висновки. Наявність в'язкої рідини призводить до зниження значення власної частоти коливань оболонки у порівнянні з частотою її коливань з будь-якою іншою рідиною. Показано також, що зі збільшенням кількості поздовжніх ребер значення частотного параметра збільшується. Крім того, значення частоти коливань збільшується з

посиленням ортотропних властивостей циліндричної оболонки і змен-
зменшується зі збільшенням витрат рідини.

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КОЛЕБАНИЯ В ЖИДКОСТИ ПРОДОЛЬНО ПОДКРЕПЛЕННОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ, ЗАПОЛНЕННОЙ ВЯЗКОЙ ЖИДКОСТЬЮ

Исследуются свободные колебания в идеальной жидкости продольно подкрепленной ортотропной цилиндрической оболочки, заполненной вязкой жидкостью. Для описания движения внутренней вязкой жидкости используется линеаризованное уравнение Навье – Стокса, движение внешней жидкости описывается волновым уравнением, записанным в потенциале скоростей возмущенного течения. На основе принципа стационарности действия Гамильтона – Остроградского получено частотное уравнение продольно упрочненной ортотропной цилиндрической оболочки с вязкой жидкостью, контактирующей с идеальной жидкостью. Построены характерные кривые зависимостей.

Ключевые слова: свободные колебания; оболочка; идеальная жидкость; напряжения; жесткость; вариационный принцип.

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