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VIBRATIONS REINFORCED BY LONGITUDINAL RIBS OF AN INHOMOGENEOUS ORTHOTROPIC CYLINDRICAL PANEL CONTACTING WITH A VISCOUS-ELASTIC MEDIUM

Vibrations of an orthotropic cylindrical panel, non-uniform in thickness, supported by longitudinal ribs, lying on a linearly viscoelastic foundation, were investigated. The Hamilton – Ostrogradsky variational principle was used to find the vibration frequencies of the panel. A frequency equation is constructed, its roots are found, and the influence of physical and geometric parameters characterizing the system is studied.

Keywords: *inhomogeneous cylindrical panel; viscous-elastic medium; ridge panel; free vibrations.*

Cylindrical panels are widely used in modern technology, power engineering, and in various fields of construction and engineering. In many cases, depending on production technology and a number of various reasons, mechanical properties of the material of cylindrical panels become continuously inhomogeneous along the length of the panel. In operational conditions these panels are in contact with different nature medium and they are stiffened when it is necessary.

In [5], a problem of lateral vibrations of a circular cross-section inhomogeneous cylindrical shell lying on a viscous-elastic foundation, is considered. It is assumed that the modulus of elasticity and density are continuous functions of thickness coordinate. The problem of natural vibrations of a circular cross-section cylindrical shell inhomogeneous only along the length and lying on inhomogeneous viscous-elastic medium, is considered in [6, 7]. The solution of the problem is reduced to the system of two linear differential equations with respect to the stress function and deflection. The method of separation of variables and the Bubnov – Galerkin method is used when solving the problem.

The paper [7] is devoted to vibrations of an orthotropic, cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linearly viscous-elastic foundation. Using the Hamilton – Ostrogradsky variational principle for finding vibrational frequencies of a cylindrical panel inhomogeneous in thickness, stiffened with lateral ribs and lying on a linear elastic foundation, the frequency equation was constructed, its roots were found and the influences of physical and geometrical parameters characterizing the system, were studied. The paper [2] was devoted to free vibrations of a flowing fluid-contacting, isotropic, inhomogeneous cylindrical shell stiffened with cross system of ribs. Using the Hamilton – Ostrogradsky variational principle,

the system of equations of motion for a flowing fluid contacting anisotropic cylindrical shell inhomogeneous in thickness and stiffened with cross systems of ribs was solved. The paper [4] deals with natural vibrations of a soil-contacting cylindrical shell stiffened with annular ribs and subjected to compressive forces.

To apply the Hamilton – Ostrogradsky variational principle, we write the total energy of the structure under investigation, consisting of an orthotropic cylindrical panel inhomogeneous in thickness and stiffening elements whose number varies. Furthermore, from the inside, the structure is in contact with a viscous-elastic medium (Fig. 1). To take into account inhomogeneity of a cylindrical shell in thickness, we will proceed from three-dimensional functional. In this case, the functional of total energy of the cylindrical shell is of the form

$$V = \frac{R}{2} \iint \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \rho(z) \left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2) dx d\phi dz. \quad (1)$$

Fig. 1 – Vibrations reinforced by longitudinal ribs of an inhomogeneous orthotropic cylindrical panel contacting with an elastic medium

There are various ways for taking into account inhomogeneity of the shell material. One of them is that the Young modulus and density of the material are accepted as functions of the normal coordinate z [3]. It is supposed that the Poisson ratio is constant. In this case, the strain-stress ratio is of the form

$$\begin{aligned} \sigma_{11} &= b_{11}(z)\varepsilon_{11} + b_{12}(z)\varepsilon_{22}; & \sigma_{22} &= b_{12}(z)\varepsilon_{11} + b_{22}(z)\varepsilon_{22}; \\ \sigma_{12} &= b_{66}(z)\varepsilon_{12} \end{aligned} \quad (2)$$

$$\varepsilon_{11} = \frac{\partial u}{\partial x}; \quad \varepsilon_{22} = \frac{\partial \vartheta}{\partial y} + w; \quad \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x}. \quad (3)$$

In (1) we can write:

$$V = \frac{R}{2} \iint \left\{ \tilde{b}_{11} \varepsilon_{11}^2 + 2\tilde{b}_{12} \varepsilon_{11} \varepsilon_{22} + 2\tilde{b}_{26} \varepsilon_{12} \varepsilon_{22} + 2\tilde{b}_{16} \varepsilon_{11} \varepsilon_{12} + \right. \\ \left. + \tilde{b}_{22} \varepsilon_{22}^2 + \tilde{b}_{66} \varepsilon_{12}^2 \right\} dx d\phi + \iint \left(\tilde{\rho} \left(\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial \vartheta}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) \right) dx d\phi, \quad (4)$$

where

$$\tilde{b}_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b_{11}(z) dz; \quad \tilde{b}_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b_{12}(z) dz; \quad \tilde{b}_{22} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b_{22}(z) dz; \quad \tilde{b}_{66} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b_{66}(z) dz; \\ b_{11}(z) = \frac{E_1(z)}{1 - \nu_1 \nu_2}; \quad b_{22}(z) = \frac{E_2(z)}{1 - \nu_1 \nu_2}; \quad b_{66}(z) = G_{12}(z) = G(z); \\ b_{12}(z) = \frac{\nu_2 E_1(z)}{1 - \nu_1 \nu_2} = \frac{\nu_1 E_2(z)}{1 - \nu_1 \nu_2}$$

are the main module of elasticity of the orthotropic material $\tilde{\rho} = \int_{-h}^h \rho(z) dz$,

ν_1, ν_2 are Poisson ratios of the orthotropic material, h is shell thickness, u, ϑ, w are the components of displacements of the points of the median surface of the shell. It is assumed that

$$E_1(z) = \tilde{E}_1 f(z), \quad E_2(z) = \tilde{E}_2 f(z), \quad G(z) = \tilde{G} f(z).$$

The expressions for the potential energy of elastic deformation of i -th lateral rib are as follows [1]:

$$\Pi_i = \frac{1}{2} \int_0^l \left[\tilde{E}_i F_i \left(\frac{\partial u_i}{\partial x} \right)^2 + \tilde{E}_i J_{yi} \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 + \tilde{E}_i J_{zi} \left(\frac{\partial^2 \vartheta_i}{\partial x^2} \right)^2 + \tilde{G}_i J_{kpi} \left(\frac{\partial \varphi_{kpi}}{\partial x} \right)^2 \right] dx. \quad (5)$$

The kinetic energy of ribs are written in the form [8]

$$K_i = \rho_i F_i \int_0^l \left[\left(\frac{\partial u_i}{\partial t} \right)^2 + \left(\frac{\partial \vartheta_i}{\partial t} \right)^2 + \left(\frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left(\frac{\partial \varphi_{kpi}}{\partial t} \right)^2 \right] dx. \quad (6)$$

In expressions (5), (6) u_i, ϑ_i, w_i are the displacements of the points of the bars used in reinforcement; F_i are the cross-sectional areas of the i -th bar,

attached to the shell in the direction of the geneatrix; $\tilde{E}_i$ are the modulus of elasticity in tension i -th bar, attached to the cylindrical shell in the direction geneatrix; J_{yi}, J_{zi} are inertia moment i -th bar relative to the axis, passing through the center of gravity of the cross-section; J_{kpi} are torsional moments of inertia i -th bar, t – time, ρ_i is density of materials of the i -th bar.

Potential energy of external surface loads acting as viewed from elastic medium, applied to the shell is determined as a work performed by these loads when taking the system from the deformed state to the initial not deformed one and is represented as follows

$$A_0 = -R \int_0^l \int_0^{\phi_0} q_z w dx d\phi. \quad (7)$$

Suppose that the plate lies on viscoelastic base, where reaction q_z is connected with flexure w in the following relation:

$$q_z = k_v w - k_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - \int_0^t \Gamma(t - \tau) w(\tau) d\tau, \quad (8)$$

where k_v is Pasternak coefficient and k_p Winkler's coefficients. The $\Gamma(t) = Ae^{-\psi t}$ – relaxation core, A, ψ – empirical constants, t – time.

The total energy of the system equals the sum of energies of elastic deformations of the shell and lateral ribs, and also potential energies of all external loads acting as viewed from viscous-elastic medium

$$W = V + \sum_{j=1}^{k_2} (\Pi_j + K_j) + A_0 \quad (9)$$

Let the plate be comprehensively fixed with hinges. Then the following boundary conditions should be fulfilled:

$$\begin{aligned} u = v = w = M_x = 0 \quad \text{for } x = 0; L, \\ u = v = w = M_x = 0 \quad \text{for } \phi = 0; \phi_0. \end{aligned} \quad (10)$$

The frequency equation of a ridge, inhomogeneous, orthotropic, flowing-fluid contacting shell was obtained on the base of Ostrogradsky – Hamilton principle of stationarity of action

$$\delta W = 0, \quad (11)$$

where $W = \int_{t'}^{t''} J dt$ is Hamilton action, t' and t'' are the given arbitrary moments of time.

Complementing the total energy of the system (9) with contact (10), we get a problem of natural vibrations of a viscous-elastic medium-contacting orthotropic cylindrical shell inhomogeneous in thickness and stiffened with lateral system of ribs. In other words, the problem of natural vibrations of a viscous-elastic medium-contacting orthotropic, cylindrical shell inhomogeneous in thickness and stiffened with cross-system of ribs is reduced to integration of expressions for the total energy of the system (9)

In expression (9) u, ϑ, w_r are variable values. These unknown values are approximated in the following way:

$$\begin{aligned} u &= u_0 \sin \frac{\pi m x}{l} \sin k \frac{\pi \varphi}{\varphi_0} \sin \omega t; & \vartheta &= \vartheta_0 \sin \frac{\pi m x}{l} \sin k \frac{\pi \varphi}{\varphi_0} \sin \omega t; \\ w &= w_0 \sin \frac{\pi m x}{l} \sin k \frac{\pi \varphi}{\varphi_0} \sin \omega t. \end{aligned} \quad (12)$$

Using (12) for w , we can calculate the work A_0 when transferring the system from the deformed state to the initial undeformed state

$$\begin{aligned} A_0 &= -R \left[\frac{\omega l}{4(\omega^2 + \psi^2)} \left(\sin \omega t - e^{\psi t} \sin^2 \omega t + \frac{\psi}{\omega} e^{\psi t} \sin^2 \omega t \right) + \right. \\ &\quad \left. + \frac{k_9 \varphi_0 l \sin^2 \omega t}{4} + k_p \left(\frac{m^2 \pi^2}{l^2} + \frac{k^2 \pi^2}{R^2 \varphi_0^2} \right) \frac{\varphi_0 l \sin^2 \omega t}{4} \right] w_0^2. \end{aligned}$$

After integrating the expression for A_0 in time t from t' to t'' , we get

$$\begin{aligned} \int_{t'}^{t''} A_0 &= -R \left\{ \frac{l}{4(\omega^2 + \psi^2)} (\cos \omega t' - \cos \omega t'') + \left[\frac{\omega l \varphi_0}{4(\omega^2 + \psi^2)} \left(\frac{\psi}{\omega} e^{\psi t} - e^{\psi t} \right) \right. \right. \\ &\quad \left. \left. + \frac{k_9 \varphi_0 l}{4} + k_p \left(\frac{m^2 \pi^2}{l^2} + \frac{k^2 \pi^2}{R^2 \varphi_0^2} \right) \frac{\varphi_0 l}{4} \right] \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t' - \sin 2\omega t''}{4\omega} \right] \right\} w_0^2. \end{aligned} \quad (13)$$

The potential and kinetic energies of elastic deformation of the i -th longitudinal ribs are follows:

$$\begin{aligned} K_i &= \frac{\rho_i F_i l \omega^2}{2} \left\{ u_0^2 \sin^2 \frac{k \pi \varphi_i}{\varphi_0} + \left(1 + \frac{J_{kpi}}{F_i} \right) \vartheta_0^2 \sin^2 \frac{k \pi \varphi_i}{\varphi_0} + \left[\sin^2 \frac{k \pi \varphi_i}{\varphi_0} + \right. \right. \\ &\quad \left. \left. + \frac{J_{kpi}}{F_i} \left(\frac{k \pi}{l \varphi_0} \right)^2 \cos^2 \frac{k \pi \varphi_i}{\varphi_0} \right] w_0^2 + \frac{J_{kpi}}{F_i} \frac{k \pi}{l \varphi_0} \sin \frac{2k \pi \varphi_i}{\varphi_0} \vartheta_0 w_0 \right\} \sin^2 \omega t; \end{aligned}$$

$$\begin{aligned}\Pi_i = & \left(\frac{m\pi}{2}\right)^2 \cdot \frac{1}{l} \left\{ \tilde{E}_i F_i \sin^2 \frac{k\pi\varphi_i}{\varphi_0} u_0^2 + \left[\tilde{E}_i J_{yi} \left(\frac{m\pi}{l}\right)^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \right. \right. \\ & \left. \left. + \tilde{G}_i J_{kpi} \cos^2 \frac{k\pi\varphi_i}{\varphi_0} \right] w_0^2 + \left[\tilde{E}_i J_{zi} \left(\frac{m\pi}{l}\right)^2 + \frac{\tilde{G}_i J_{kpi}}{R^2} \right] \sin^2 \frac{k\pi\varphi_i}{\varphi_0} v_0^2 + \right. \\ & \left. + \frac{\tilde{G}_i J_{kpi}}{R^2} \frac{k\pi}{l\varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} \vartheta_0 w_0 \right\} \sin^2 \omega t.\end{aligned}$$

After integrating the expression for the potential and kinetic energy of elastic deformation of the i – th longitudinal ribs in time t from t' to t'' , we obtain:

$$\begin{aligned}\Pi_i = & \left(\frac{m\pi}{2}\right)^2 \cdot \frac{1}{l} \left\{ \tilde{E}_i F_i \sin^2 \frac{k\pi\varphi_i}{\varphi_0} u_0^2 + \left[\tilde{E}_i J_{yi} \left(\frac{m\pi}{l}\right)^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \right. \right. \\ & \left. \left. + \tilde{G}_i J_{kpi} \cos^2 \frac{k\pi\varphi_i}{\varphi_0} \right] w_0^2 + \left[\tilde{E}_i J_{zi} \left(\frac{m\pi}{l}\right)^2 + \frac{\tilde{G}_i J_{kpi}}{R^2} \right] \sin^2 \frac{k\pi\varphi_i}{\varphi_0} v_0^2 + \right. \\ & \left. + \frac{\tilde{G}_i J_{kpi}}{R^2} \frac{k\pi}{l\varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} \vartheta_0 w_0 \right\} \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t'' - \sin 2\omega t'}{4\omega} \right]; \\ K_i = & \frac{\rho_i F_i l \omega^2}{2} \left\{ u_0^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \left(1 + \frac{J_{kpi}}{F_i} \right) \times \vartheta_0^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \left[\sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \right. \right. \\ & \left. \left. + \frac{J_{kpi}}{F_i} \left(\frac{k\pi}{l\varphi_0} \right)^2 \cos^2 \frac{k\pi\varphi_i}{\varphi_0} \right] w_0^2 + \frac{J_{kpi}}{F_i} \frac{k\pi}{l\varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} \vartheta_0 w_0 \right\} \times \\ & \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t'' - \sin 2\omega t'}{4\omega} \right].\end{aligned}\tag{14}$$

Using (12) for w , we can calculate the total energy V of the cylindrical shell

$$\begin{aligned}V = & \frac{lR\varphi_0}{8} \left\{ \left[\tilde{b}_{11} \left(\frac{m\pi}{l}\right)^2 + 2\tilde{b}_{66} \right] u_0^2 + \left[\tilde{b}_{22} \left(\frac{k\pi}{\varphi_0 R}\right)^2 + \tilde{b}_{66} \left(\frac{m\pi}{l}\right)^2 \right] \vartheta_0^2 + \right. \\ & \left. + \frac{\tilde{b}_{22} l \varphi_0}{4} w_0^2 + 4 \left[\tilde{b}_{26} \left(\frac{k\pi}{\varphi_0 R}\right)^2 + \tilde{b}_{16} \left(\frac{m\pi}{l}\right)^2 \right] u_0 \vartheta_0 \right\} \sin^2 \omega t.\end{aligned}$$

After integrating the expression for the total energy V of the cylindrical shell: in time t from t' to t'' , we obtain:

$$\begin{aligned}
V = & \frac{IR\varphi_0}{8} \left\{ \left[\tilde{b}_{11} \left(\frac{m\pi}{l} \right)^2 + 2\tilde{b}_{66} \right] u_0^2 + \left[\tilde{b}_{22} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{66} \left(\frac{m\pi}{l} \right)^2 \right] \vartheta_0^2 + \right. \\
& + \frac{\tilde{b}_{22} l \varphi_0}{4} w_0^2 + 4 \left[\tilde{b}_{26} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{16} \left(\frac{m\pi}{l} \right)^2 \right] u_0 \vartheta_0 \left. \right\} \times \\
& \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t' - \sin 2\omega t''}{4\omega} \right].
\end{aligned} \tag{15}$$

Using the expressions (13), (14) and (15) for the complete energy of the system under study, we obtain:

$$\begin{aligned}
W = & \left\{ \frac{IR\varphi_0}{8} \left[\tilde{b}_{11} \left(\frac{m\pi}{l} \right)^2 + 2\tilde{b}_{66} \right] + \left(\frac{m\pi}{2} \right)^2 \frac{1}{l} \sum_{i=1}^{k_1} \left(\tilde{E}_i F_i \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \right. \right. \\
& + \left. \frac{\rho_i F_i l \omega^2}{2} \sin^2 \frac{k\pi\varphi_i}{\varphi_0} \right) \left. \right\} u_0^2 + \left\{ \frac{IR\varphi_0}{8} \left[\tilde{b}_{22} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{66} \left(\frac{m\pi}{l} \right)^2 \right] + \right. \\
& + \left(\frac{m\pi}{2} \right)^2 \cdot \frac{1}{l} \sum_{i=1}^{k_1} \left[\tilde{E}_i J_{zi} \left(\frac{m\pi}{l} \right)^2 + \frac{\tilde{G}_i J_{kpi}}{R^2} + \frac{\rho_i F_i l \omega^2}{2} \left(1 + \frac{J_{kpi}}{F_i} \right) \right] \sin^2 \frac{k\pi\varphi_i}{\varphi_0} \left. \right\} \vartheta_0^2 + \\
& + \left\{ \frac{\tilde{b}_{22} l \varphi_0}{4} + \left(\frac{m\pi}{2} \right)^2 \cdot \frac{1}{l} \sum_{i=1}^{k_1} \left[\tilde{E}_i J_{yi} \left(\frac{m\pi}{l} \right)^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \tilde{G}_i J_{kpi} \cos \frac{2k\pi\varphi_i}{\varphi_0} \right] - \right. \\
& - \frac{Rl}{4(\omega^2 + \psi^2)} (\cos \omega t' - \cos \omega t'') \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t' - \sin 2\omega t''}{4\omega} \right]^{-1} - \\
& - R \left[\frac{\omega l \varphi_0}{4(\omega^2 + \psi^2)} \left(\frac{\psi}{\omega} e^{\psi t} - e^{\psi t'} \right) + \frac{k_9 \varphi_0 l}{4} + k_p \left(\frac{m^2 \pi^2}{l^2} + \frac{k^2 \pi^2}{R^2 \varphi_0^2} \right) \frac{\varphi_0 l}{4} \right] \left. \right\} w_0^2 + \\
& + \frac{IR\varphi_0}{2} \left[\tilde{b}_{26} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{16} \left(\frac{m\pi}{l} \right)^2 \right] u_0 \vartheta_0 + \\
& + \left[\left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \frac{\tilde{G}_i J_{kpi}}{R^2} \frac{k\pi}{l^2 \varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} + \frac{k\pi}{\varphi_0} \sum_{i=1}^{k_1} \frac{\rho_i F_i \omega^2}{2} \frac{J_{kpi}}{F_i} \sin \frac{2k\pi\varphi_i}{\varphi_0} \right] \vartheta_0 w_0 \left. \right\} \times \\
& \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t' - \sin 2\omega t''}{4\omega} \right].
\end{aligned}$$

Substituting (16) in (9), after integration we get a function of variables u_0, ϑ_0, w_0 . The stationary value of the obtained function is determined by the following system

$$1) \frac{\partial W}{\partial u_0} = 0; \quad 2) \frac{\partial W}{\partial \vartheta_0} = 0; \quad 3) \frac{\partial W}{\partial w_0} = 0. \quad (17)$$

The non-trivial solution of the system of third order linear algebraic equations is possible only in the case when ω is the root of its determinant. The definition of ω reduces to the algebraic equation

$$\det \|a_{ij}\| = 0, i, j = 1, 3 \quad (18)$$

Here the elements a_{ij} look like:

$$\begin{aligned} a_{11} &= \frac{lR\varphi_0}{4} \left[\tilde{b}_{11} \left(\frac{m\pi}{l} \right)^2 + 2\tilde{b}_{66} \right] + \frac{2}{l} \left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \left(\tilde{E}_i F_i \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \frac{\rho_i F_i l \omega^2}{2} \sin^2 \frac{k\pi\varphi_i}{\varphi_0} \right); \\ a_{12} &= \frac{lR\varphi_0}{2} \left[\tilde{b}_{26} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{16} \left(\frac{m\pi}{l} \right)^2 \right]; \quad a_{13} = 0; \\ a_{21} &= \frac{lR\varphi_0}{2} \left[\tilde{b}_{26} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{16} \left(\frac{m\pi}{l} \right)^2 \right]; \\ a_{22} &= \frac{lR\varphi_0}{4} \left[\tilde{b}_{22} \left(\frac{k\pi}{\varphi_0 R} \right)^2 + \tilde{b}_{66} \left(\frac{m\pi}{l} \right)^2 \right] + \\ &\quad + \frac{2}{l} \left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \left[\tilde{E}_i J_{zi} \left(\frac{m\pi}{l} \right)^2 + \frac{\tilde{G}_i J_{kpi}}{R^2} + \frac{\rho_i F_i l \omega^2}{2} \left(1 + \frac{J_{kpi}}{F_i} \right) \sin^2 \frac{k\pi\varphi_i}{\varphi_0} \right]; \\ a_{23} &= \left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \frac{\tilde{G}_i J_{kpi}}{R^2} \frac{k\pi}{l^2 \varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} + \frac{k\pi}{\varphi_0} \sum_{i=1}^{k_1} \frac{\rho_i F_i \omega^2}{2} \frac{J_{kpi}}{F_i} \sin \frac{2k\pi\varphi_i}{\varphi_0}; \\ a_{31} &= 0; \quad a_{32} = \left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \frac{\tilde{G}_i J_{kpi}}{R^2} \frac{k\pi}{l^2 \varphi_0} \sin \frac{2k\pi\varphi_i}{\varphi_0} + \frac{k\pi}{\varphi_0} \sum_{i=1}^{k_1} \frac{\rho_i F_i \omega^2}{2} \frac{J_{kpi}}{F_i} \sin \frac{2k\pi\varphi_i}{\varphi_0}; \\ a_{33} &= \frac{\tilde{b}_{22} l \varphi_0}{2} + \frac{2}{l} \left(\frac{m\pi}{2} \right)^2 \sum_{i=1}^{k_1} \left[\tilde{E}_i J_{yi} \left(\frac{m\pi}{l} \right)^2 \sin^2 \frac{k\pi\varphi_i}{\varphi_0} + \tilde{G}_i J_{kpi} \cos^2 \frac{k\pi\varphi_i}{\varphi_0} \right] - \\ &\quad - \frac{Rl}{2(\omega^2 + \psi^2)} (\cos \omega t' - \cos \omega t'') \times \left[\frac{1}{2} (t'' - t') - \frac{\sin 2\omega t' - \sin 2\omega t''}{4\omega} \right]^{-1} - \\ &\quad - 2R \left[\frac{\omega l \varphi_0}{4(\omega^2 + \psi^2)} \left(\frac{\psi}{\omega} e^{\psi t} - e^{\psi t} \right) + \frac{k_9 \varphi_0 l}{4} + k_p \left(\frac{m^2 \pi^2}{l^2} + \frac{k^2 \pi^2}{R^2 \varphi_0^2} \right) \frac{\varphi_0 l}{4} \right]. \end{aligned}$$

Equation (18) was calculated by the numerical method. The parameters contained in the solution of the problem were accepted as:

$$\begin{aligned}\rho_0 &= \rho_j = 18,5 \cdot 10^3 \text{ H / m}^3, \quad \tilde{E}_j = 6,67 \cdot 10^9 \text{ N / m}^2; \\ m &= 1; n = 8; h_j = 1,39 \cdot 10^{-3} \text{ m}; R = 1,6 \text{ m}; \quad F_i = 5,2 \cdot 10^{-6} \text{ m}^2; \\ I_{kpj} &= 0,23 \cdot 10^{-12} \text{ m}^4; I_{xj} = 5,1 \cdot 10^{-12} \text{ m}^4; I_{zi} = 1,3 \cdot 10^{-12} \text{ m}^4, \quad \nu = 0.35; \\ \frac{l}{R} &= 3, \quad \frac{h}{R} = \frac{1}{6}, \quad f(z) = 1 + \varepsilon \left(\frac{z}{h} \right)^2, \quad \varepsilon \in [0; 1], \\ k_v &= 10^6 \text{ N / m}^3, \quad k_p = 10^4 \text{ N / m}, \quad A = \psi \\ \rho_0 &= \rho_j = 1850 \text{ kg / m}^3, \quad \tilde{E}_j = 6.67 \cdot 10^9 \text{ N / m}^2; \\ m &= 1; n = 8; h_j = 1,39; R = 160 \text{ sm}; \quad F_i = 5.2 \text{ mm}^2 \\ I_{kpj} &= 0.23 \text{ mm}^4; I_{xj} = 5.1 \text{ mm}^4; I_{zi} = 1.3 \text{ mm}^4, \quad \nu = 0.35;\end{aligned}$$

The result of calculations were given in Fig. 2 in the form of dependence of the frequency ω on the amount of stiffening bars k_2 on the shell surface, in Fig. 3 in the form of dependence of frequency ω on in homogeneity parameter ε .

Fig. 2 – Dependence of vibrations frequency ω of the system from the amount of ribs k_2

Fig. 3 – Dependence of vibrations frequency ω of the system from the inhomogeneity parameter ε

The figure shows that with increasing the amount of lateral ribs, the value of the vibrations frequency of the system increases. Fig. 3 shows that with increasing the inhomogeneity parameters the vibrations frequencies of the system also increase.

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КОЛИВАННЯ НЕОДНОРІДНОЇ ОРТОТРОПНОЇ ЦИЛІНДРИЧНОЇ ПАНЕЛІ, ПІДКРІПЛЕНОЇ ПОЗДОВЖНИМИ РЕБРАМИ, ЩО КОНТАКТУЄ З В'ЯЗКО-ПРУЖНИМ СЕРЕДОВИЩЕМ

Досліджено коливання неоднорідної по товщині, підкріпленої поздовжніми ребрами, ортотропної циліндричної панелі, що лежить на лінійно в'язко-пружній основі. Застосовано варіаційний принцип Гамільтона – Остроградського для знаходження частот коливань панелі. Побудовано частотне рівняння, знайдені його коріння і вивчено вплив фізичних та геометричних параметрів, що характеризують систему.

Ключові слова: неоднорідні циліндричні панелі; в'язко-пружне середовище; ребриста панель; віпні коливання.

Циліндричні панелі широко використовуються в сучасній техніці, енергетиці та в різних областях будівництва і машинобудуванні. У багатьох випадках в залежності від технології виготовлення і ряду інших причин механічні властивості матеріалу циліндричної панелі стають неоднорідними по її довжині. У робочих умовах ці панелі контактують із середовищем різної природи і при необхідності підкріплюються. Для урахування неоднорідності матеріалу оболонки по товщині приймається, що модуль Юнга і щільність матеріалу оболонки є функціями нормальної координати z .

З використанням варіаційного принципу Гамільтона – Остроградського записується загальна енергія досліджуваної конструкції, що складається з ортотропної циліндричної панелі, неоднорідної за товщиною, та елементів жорсткості, кількість яких змінюється. Щоб врахувати неоднорідність циліндричної оболонки по товщині, ми виходитимемо з тривимірного функціоналу.

Потенціальна енергія зовнішніх поверхневих навантажень, що діють з огляду на еластичне середовище, прикладених до оболонки, визначається як робота, яку виконують ці навантаження при переведенні системи з деформованого стану в початковий недеформований. Рівняння частоти ребра панелі, неоднорідної, ортотропної, що контактує з текучою рідиною, було отримано на основі принципу стаціонарності Гамільтона – Остроградського. В процесі обчислень для функції неоднорідності прийняті параболічні закони.

Побудовано характерні залежності для неоднорідної по товщині підкріпленої поздовжніми ребрами циліндричної панелі, що лежить на лінійно в'язко-пружній основі. Показано, що зі збільшенням кількості бічних ребер значення частоти коливань системи зростає, а зі збільшенням параметрів неоднорідності також зростають частоти коливань системи.

А. Х. Мовсумова

**КОЛЕБАНИЯ НЕОДНОРОДНОЙ ОРТОТРОПНОЙ
ЦИЛИНДРИЧЕСКОЙ ПАНЕЛИ, ПОДКРЕПЛЕННОЙ
ПРОДОЛЬНЫМИ РЕБРАМИ И КОНТАКТИРУЮЩЕЙ
С ВЯЗКО-УПРУГОЙ СРЕДОЙ**

Исследованы колебания неоднородной по толщине, подкрепленной продольными ребрами, ортотропной цилиндрической панели, лежащей на линейно вязко-упругом основании. Использован вариационный принцип Гамильтона – Остроградского для нахождения частот колебаний панели. Построено частотное уравнение, найдены его корни и изучено влияние физических и геометрических параметров, характеризующих систему.

Ключевые слова: неоднородные цилиндрические панели; вязко-упругая среда; ребристая панель; свободные колебания.

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